We now know how to calculate decay rate Γ and cross section σ in terms of the amplitude $\mathcal{M}$

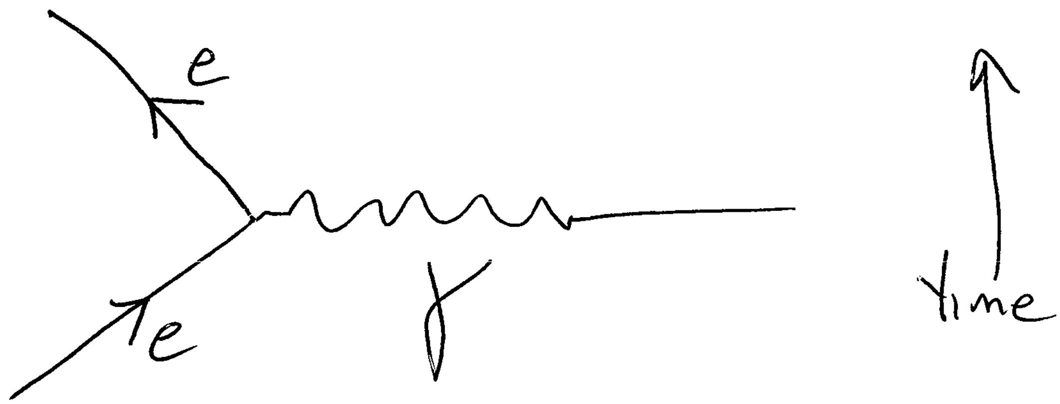
Now we need to know the value of $\mathcal{M}$!

This is what the Feynman diagrams were invented for.

Originally intended for use as a "bookkeeping device" for QED calculations.

We now use them to represent almost any particle interactions graphically.

In QED the basic diagram is the primitive vertex with electrons and photons interacting:



This looks simple but there are a couple of things that complicate it and distract us from learning how to use it.

- electrons have spin 1/2, photon spin 1
- photons are massless

We'll get into that later.

J. Hopkins

To learn the method let's consider a model - does not represent the real world but works the same way.

Toy system

3 particles

A spin 0 mass M_A

B spin 0 mass M_B

C spin 0 mass M_C

A is heavier than B + C so it can decay

$A \to B + C$, $M_A > M_B + M_C$

$\bar{A} = A$, $\bar{B} = B$, $\bar{C} = C$

all are their own antiparticle

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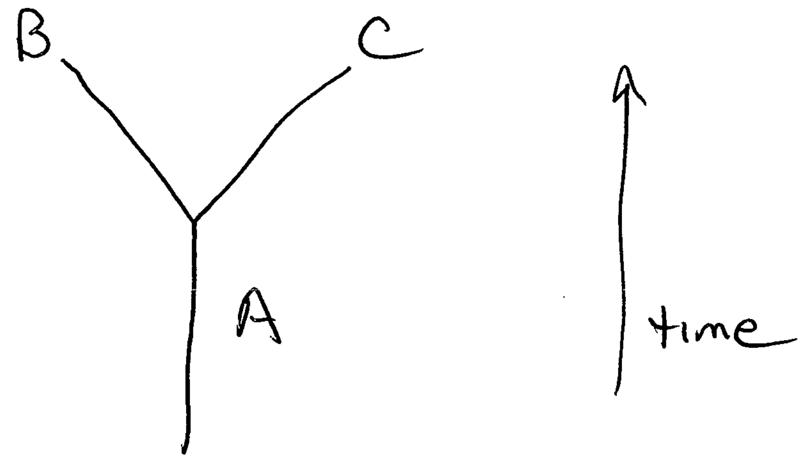
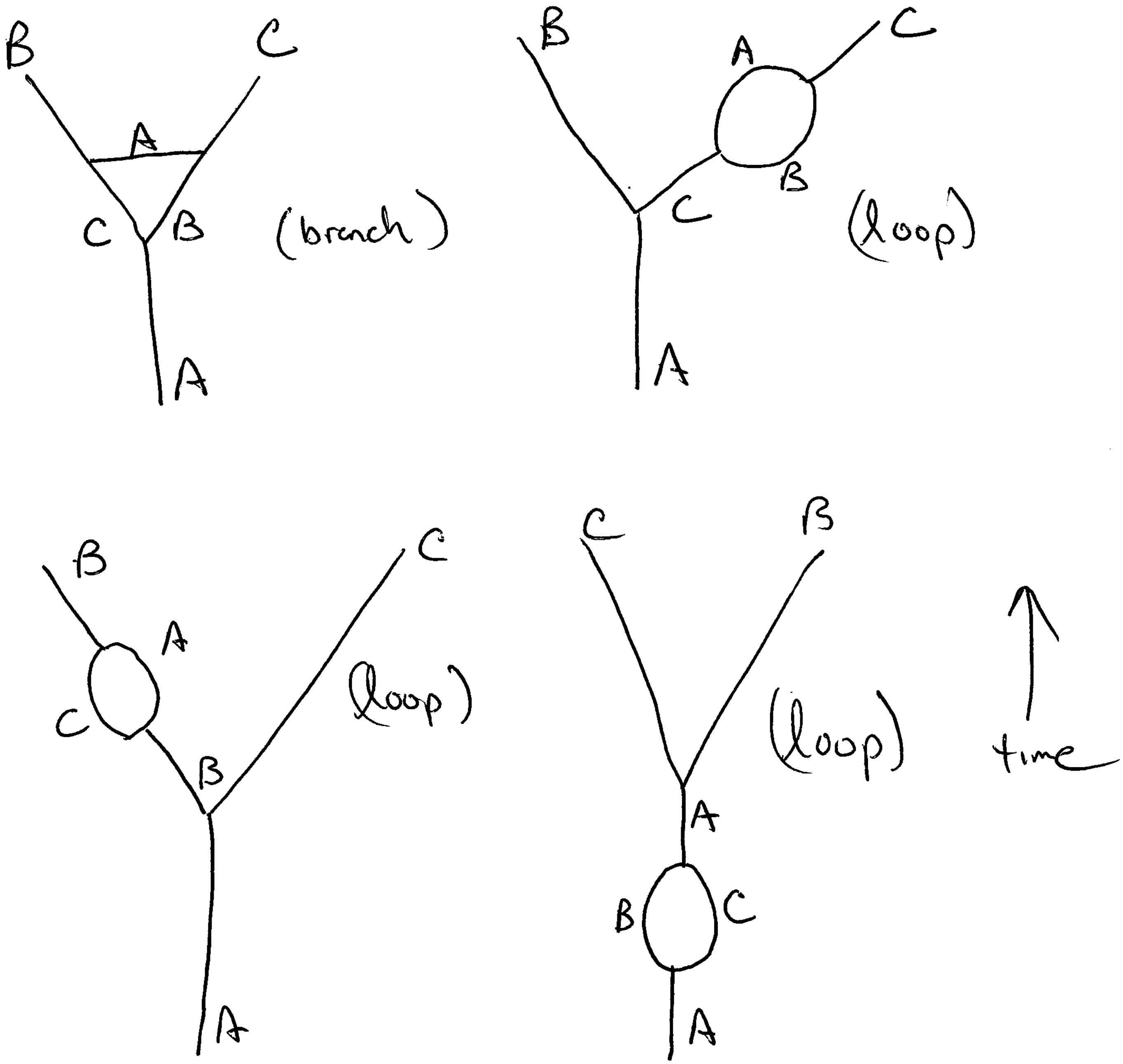
The primitive vertex for this system looks like this:



(since all are their own antiparticle, we don't need arrows)

In addition to the primitive vertex, there can be higher order diagrams:



and even higher order diagrams with more loops and branches. Of course same situation holds for real Feynman diagrams.

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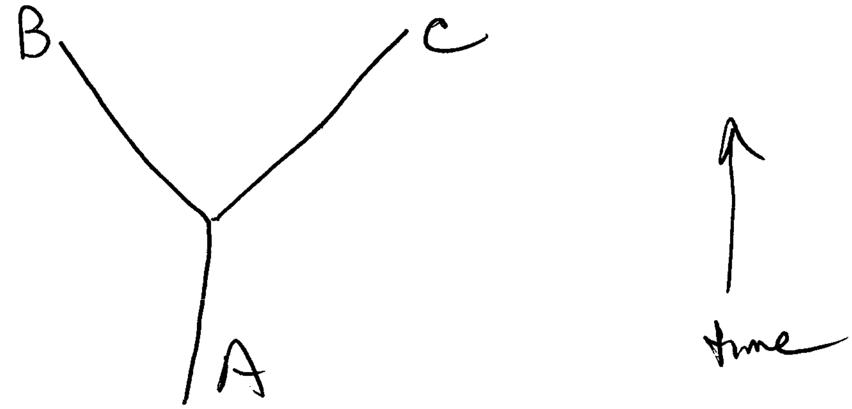
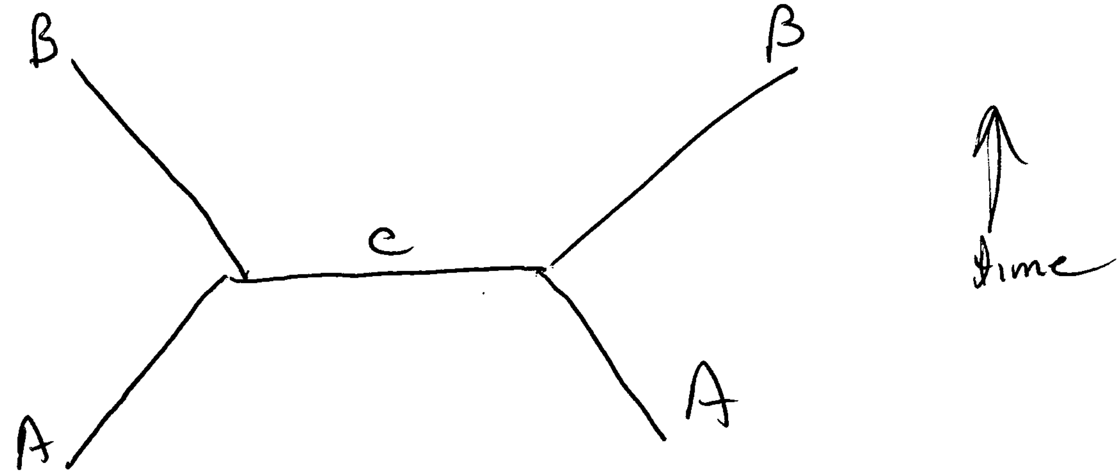
First we will look at the decay of the A (to lowest order)



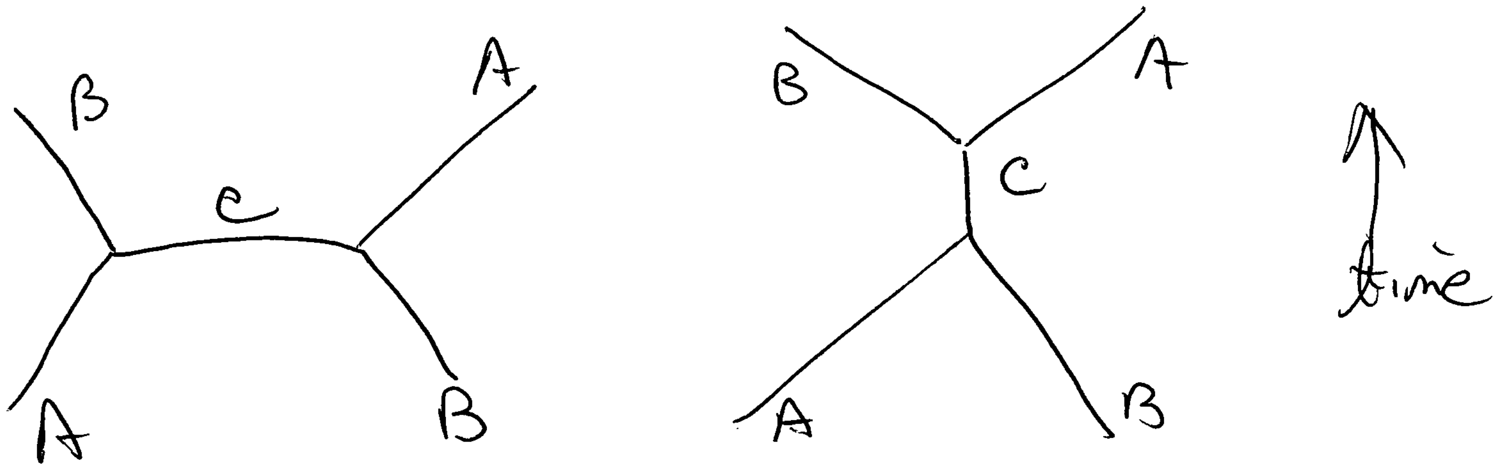
Then at some scattering processes

A + A ⟶ B + B



(Möller)

or $A + B \to A + B$



(Bhabha)

So I'm going to give you the rules and then we'll calculate the A decay.

Griffiths gives us a recipe for calculating $\mathcal{M}$ from a Feynman diagram.

1) Label everything.

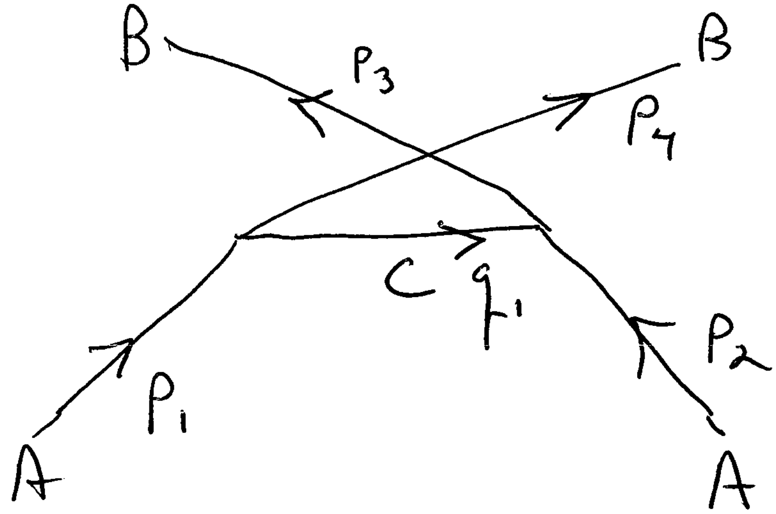
Incoming and outgoing lines (representing 4-momenta) are labeled with $p_1, p_2 \ldots p_n$ and get an arrow indicating direction in time.

Internal 4-momenta lines are labeled $q_1, q_2 \ldots q_n$ and get an arbitrary direction arrow

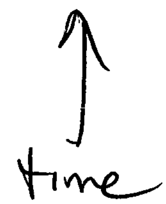
Griffiths gives an example of a 'generic Feynman diagram' but it just looks like an egg with lines.

8 J. Hopkins 8

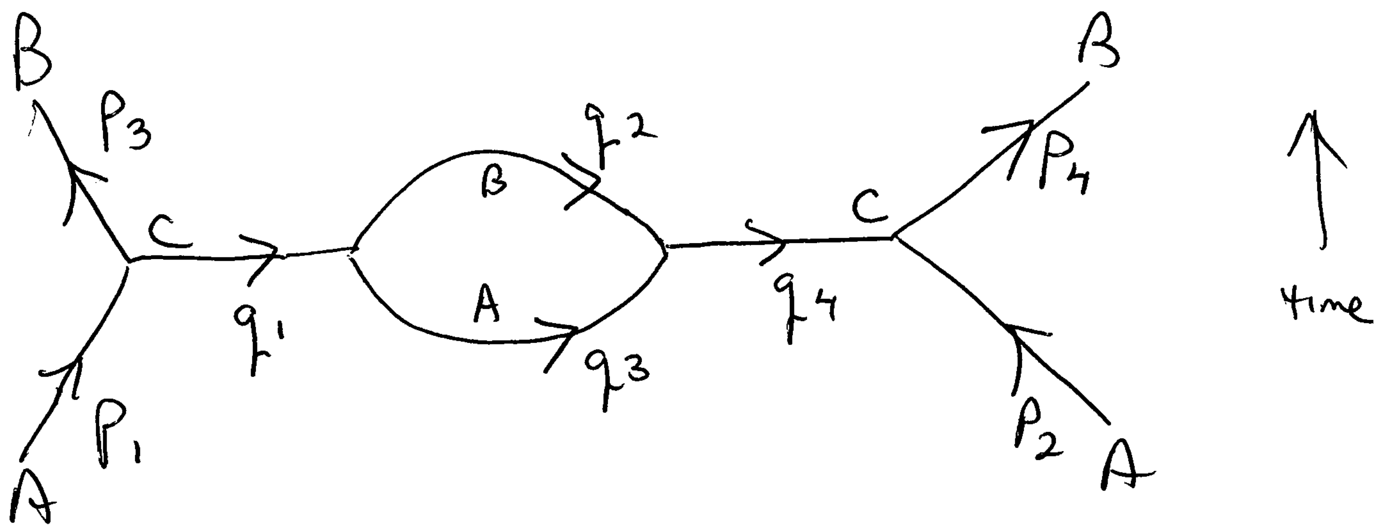
Here's a better example which is Figure 6.9 (6.8 in 1st Edition)



$A + A \to B + B$



or in Section 6.3.3 (6.8 in 1st Edition)



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2) For each vertex, write down a coupling factor of $-ig$ - this specifies the strength of the interaction. In the toy theory of ABC, this (g) will have dimensions of momentum. In the real QED, it's dimensionless. The significance of this is that if it's a small number (< 0) like say, $\frac{1}{137}$ then higher order diagrams with more vertices will contribute much less to final value.

J Hopkins 10

3) Propagators – for each <u>internal</u> line, write a factor of:

$$\frac{i}{q_j^2 - m_j^2 c^2}$$

where q_j is the 4-momentum of the particle's line and m_j is the particle's mass.

$q_j^2 \neq m_j^2 c^2$ because a virtual particle does not lie on its mass shell. (All that means is that a virtual particle doesn't have the same mass as the free particle – in fact it can have any mass)

4) Conserve Energy and Momentum.

For each vertex, write a delta function

$$(2\pi)^4 \delta^4(k_1 + k_2 + k_3)$$

the k's are the momenta of the lines in and out of the vertex - use (−) sign for outgoing lines

since $\delta(x) = \begin{bmatrix} 0 \text{ if } x \neq 0 \\ \infty \text{ if } x = 0 \end{bmatrix}$

then this will be 0 unless the sum of the incoming and outgoing momenta is 0

example $\delta^4(k_1 + k_2 - k_3)$

5) Integrate over the internal momenta

For each internal line, write a factor of

$$\frac{1}{(2\pi)^4} d^4 q_i$$

and then integrate over all the internal momenta

so you'll have something that looks like:

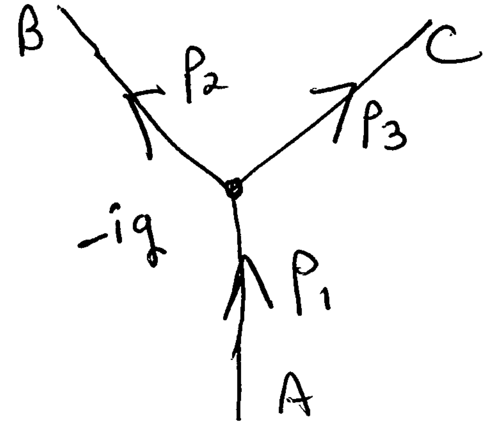
$$\int -ig \left(\frac{i}{q_1^2 - m_1^2 c^2}\right)(2\pi)^4 \delta^4(p_1 + p_2 - p_3) \frac{1}{(2\pi)^4} d^4 q_1$$

when you've integrated that, you'll have delta function in the result.

erase it and multiply by i

The result is .... $\mathcal{M}$!

Now let's calculate the lifetime of the A:



no internal lines so we get

$$-ig(2\pi)^4\delta^4(p_1-p_2-p_3)$$

no internal momenta, so nothing to integrate.

discard the delta function and multiply by i (the $(2\pi)^4$ is part of the delta function)

we get $i(-ig)=g$.

$$\mathcal{M}=g$$

J Hopkins (14)

plug $\mathcal{M}$ into equation 6.35
(6.32 in 1st Edition)

$$\Gamma = \frac{g^2 |p|}{8\pi\hbar m_A^2 c}$$

$|p|$ is the momentum of either of the products of the decay

E_A, since A was at rest, is

$$E_A = m_A c^2$$

After the decay, E is

$$E = \sqrt{m_B^2 c^4 + p^2 c^2} + \sqrt{m_C^2 c^4 + p^2 c^2}$$

Solving for P gives us

$$|p| = \frac{c}{2m_A}\sqrt{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$$

(see attached)

J. Hopkins 15

The lifetime of the A is

$$\tau = \frac{1}{\Gamma} = \frac{8\pi \hbar m_A^2 c}{g^2 |p|}$$

8π - constant

$\hbar$ - E · time

$$m_A^2 - \frac{E^2}{V^4}$$

c - V

$$g^2 = \frac{E^2}{V^2}$$

$$p = \frac{E}{V}$$

$$\frac{E \cdot time \cdot \frac{E^2}{V^4} \cdot V}{\frac{E^3}{V^3}} = time$$

J. HOPKINS 16

$$E_{INIT} = m_A c^2 = E_{FINAL} = \sqrt{m_B^2 c^4 + p^2 c^2} + \sqrt{m_C^2 c^4 + p^2 c^2}$$

$$m_A c^2 = c\left[\sqrt{m_B^2 c^2 + p^2} + \sqrt{m_C^2 c^2 + p^2}\right]$$

$$m_A c = \sqrt{m_B^2 c^2 + p^2} + \sqrt{m_C^2 c^2 + p^2}$$

SQUARE BOTH SIDES

$$m_A^2 c^2 = m_B^2 c^2 + p^2 + m_C^2 c^2 + p^2 + 2\sqrt{m_B^2 c^2 + p^2}\sqrt{m_C^2 c^2 + p^2}$$

$$m_A^2 c^2 - m_B^2 c^2 - m_C^2 c^2 - 2p^2 = 2\sqrt{m_B^2 c^2 + p^2}\sqrt{m_C^2 c^2 + p^2}$$

SQUARE THEM AGAIN

$$m_A^4c^4 + m_B^4c^4 + m_C^4c^4 + 4p^4$$

$$- m_A^2c^2m_B^2c^2 - m_A^2c^2m_B^2c^2$$

$$- m_A^2c^2m_C^2c^2 - m_A^2c^2m_C^2c^2$$

$$- 2m_A^2c^2p^2 - 2m_A^2c^2p^2$$

$$+ m_B^2c^2m_C^2c^2 + m_B^2c^2m_C^2c^2$$

$$+ 2m_B^2c^2p^2 + 2m_B^2c^2p^2$$

$$+ 2m_C^2c^2p^2 + 2m_C^2c^2p^2$$

$$= 4(m_B^2c^2 + p^2) \times (m_C^2c^2 + p^2)$$

COMBINE TERMS

$$m_A^4c^4+m_B^4c^4+m_C^4c^4+4p^4$$
$$-2m_A^2m_B^2c^4$$
$$-2m_A^2m_C^2c^4$$
$$-4m_A^2p^2c^2$$
$$+2m_B^2m_C^2c^4$$
$$+4m_B^2p^2c^2$$
$$+4m_C^2p^2c^2$$

$$= 4m_B^2m_C^2c^4 + 4m_B^2p^2c^2 + 4m_C^2p^2c^2 + 4p^4$$

SIMPLIFY

$$m_A^4c^4+m_B^4c^4+m_C^4c^4$$
$$-2m_A^2m_B^2c^4$$
$$-2m_A^2m_C^2c^4$$
$$-4m_A^2p^2c^2$$

$$= 2m_B^2m_C^2c^4$$

get p on one side

$$m_A^4c^4+m_B^4c^4+m_C^4c^4$$
$$-2m_Am_Bc^4-2m_Am_Cc^4$$
$$-2m_Bm_Cc^4$$

$$= 4m_A^2p^2c^2$$

divide by $4m_A^2c^2$

$$\frac{1}{4m_A^2c^2}\left(m_A^4c^4+m_B^4c^4+m_C^4c^4-m_A^2m_B^2c^4-m_A^2m_C^2c^4-m_B^2m_C^2c^4\right)$$

$$= p^2$$

take c^4 out of the parenthesis

and take the root

$$\frac{1\cdot c^2}{2m_A c}\sqrt{m_A^4+m_B^4+m_C^4-m_Am_B-m_Am_C-m_Bm_C}$$

$$= p$$

$$= \frac{c}{2m_A}\sqrt{m_A^4+m_B^4+m_C^4-m_Am_B-m_Am_C-m_Bm_C}$$