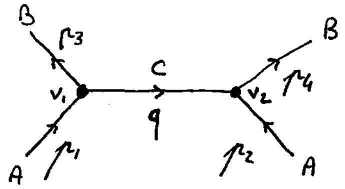
6.5 Scattering:

Apply Feynman Rules to $A + A \to B + B$ modeled by:



① LABEL the elements of the diagram

→ internal lines with q

→ outgoing and incoming by p_i

② Write a factor of $(-ig)$ for each vertex

⇒ 2 vertices → $(-ig)^2$

③ For the internal line for particle C with spin 0, write the propagator:

$$\frac{i}{q^2-(m_c c)^2}$$

④ Write δ function corresponding to each vertex

V_1: $(2\pi)^4\,\delta^4(p_1-p_3-q)$

V_2: $(2\pi)^4\,\delta^4(p_2+q-p_4)$

⑤ Integrate over all internal lines

5.1 one line only → multiply by $\frac{1}{(2\pi)^4}d^4q$

5.2 write clearly the integration to do:

$$\int (-ig)^2\frac{i}{q^2-(m_c c)^2}\times(2\pi)^4\,\delta^4(p_1-p_3-q)\,(2\pi)^4\,\delta^4(p_2+q-p_4)\frac{1}{(2\pi)^4}\,d^4q$$

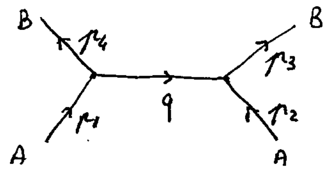
$$= -ig^2(2\pi)^4\int\frac{\delta^4(q-(p_1-p_3))\;\delta^4(q-(p_4-p_2))}{q^2-(m_c c)^2}\,d^4q$$

$$= -ig^2(2\pi)^4\,\frac{\delta^4\left((p_4-p_2)-(p_1-p_3)\right)}{(p_4-p_2)^2-(m_c c)^2}$$

⑥ Remove $(2\pi)^4\,\delta^4$ and identify with $-i\mathcal{M}$:

$$-i\mathcal{M}=\frac{-ig^2}{(p_4-p_2)^2-(m_c c)^2}\quad\Rightarrow\quad\mathcal{M}=\frac{g^2}{(p_4-p_2)^2-(m_c c)^2}$$

Another diagram for the same process is when the labels of the outgoing particles are flipped:



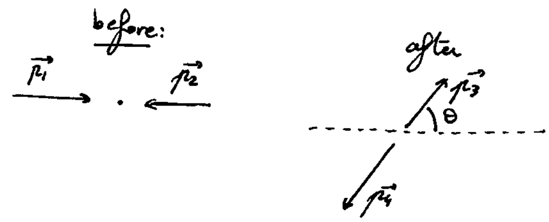
I can avoid going through the previous algebra by simply flipping p_3 & p_4 everywhere in the above derivation.

The final amplitude is then:

$$\boxed{\mathcal{M} = g^2\left(\frac{1}{(p_4-p_2)^2-(m_c c)^2} + \frac{1}{(p_3-p_2)^2-(m_c c)^2}\right)}$$

Calculate now the Differential Cross Section (DCS) in the CM frame. Assume $m_A = m_B = m$ & $m_c = 0$

$$(p_1)^2 = \left(\frac{E_1}{c}\right)^2 - |\vec{p}_1|^2 = (mc)^2 = \left(\frac{E_2}{c}\right)^2 - |\vec{p}_2|^2 .$$



In the CM frame, $\vec{p}_1 + \vec{p}_2 = \vec{0} = \vec{p}_3 + \vec{p}_4$

Therefore: $|\vec{p}_1| = |\vec{p}_2|$ & $|\vec{p}_3| = |\vec{p}_4|$

Thus $E_1 = E_2$ and likewise, $E_3 = E_4$. Conservation of energy imposes $E_1 + E_2 = E_3 + E_4$

Thus: $E_1 = E_3 = E_2 = E_4 = E$.

Particles A & B having the same rest mass yields:

$$\left(\frac{E}{c}\right)^2 - |\vec{p}_2|^2 = \left(\frac{E}{c}\right)^2 - |\vec{p}_4|^2 \implies |\vec{p}_2| = |\vec{p}_4| = p$$

Now let's evaluate $(p_4-p_2)^2$ and $(p_3-p_2)^2$:

- $$(p_4-p_2)^2 = \left(\frac{E_4-E_2}{c},\ -p\cos\theta - p,\ -p\sin\theta,\ 0\right)^2 = \left(0,\ -p(1-\cos\theta),\ -p\sin\theta,\ 0\right)^2$$
$$= 0^2 - p^2(1-\cos\theta)^2 - p^2\sin^2\theta = -p^2(1 - 2\cos\theta + \cos^2\theta + \sin^2\theta)$$
$$= -2p^2(1-\cos\theta)$$

- $$(p_3-p_2)^2 = \left(\left(\frac{E_3-E_2}{c}\right),\ p(\cos\theta+1),\ p\sin\theta,\ 0\right)^2 = -p^2\left[(\cos\theta+1)^2 + \sin^2\theta\right] = -p^2(\cos^2\theta + 1 + 2\cos\theta + \sin^2\theta)$$
$$= -2p^2(1+\cos\theta)$$

Then: $$\mathcal{M} = \frac{-g^2}{2p^2}\left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}\right) = \frac{-g^2}{2p}\left(\frac{1+\cos\theta + 1 - \cos\theta}{1-\cos^2\theta}\right) = \frac{-g^2}{2p}\left(\frac{1}{\sin^2\theta}\right)$$

Use now Eq (6.42)

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S\,|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

Here $S = \frac{1}{2!} = \frac{1}{2}$, $|\vec{p}_f| = |\vec{p}_i| = p$, $E_1 = E_2 = E$

So

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \times \frac{1}{2} \times \frac{1}{(2E)^2} \times \left|\frac{g^2}{p^2 \sin^2\theta}\right|^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{2}\left|\frac{\hbar c \; g^2}{16\pi E \; p^2 \sin^2\theta}\right|^2}$$