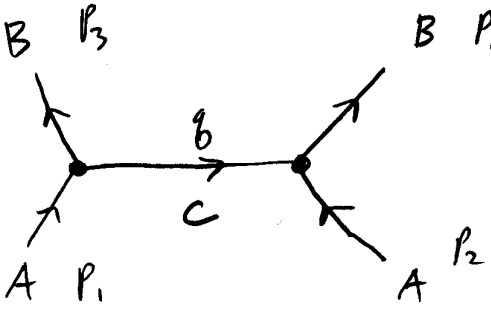
Sec. 6.5

consider scattering process

$$A + A \Rightarrow B + B$$

Lowest order diagrams include



Rule 1 — Label diagram

Apply rules:

$$-iM = \int \underset{\text{Rule 2}}{(-ig)} \; \underset{\text{Rule 3}}{\frac{i}{q^2 - m_c^2}} \; \underset{\text{Rule 2}}{(-ig)} \; (2\pi)^4 \delta^4(p_1 - p_3 - q)$$

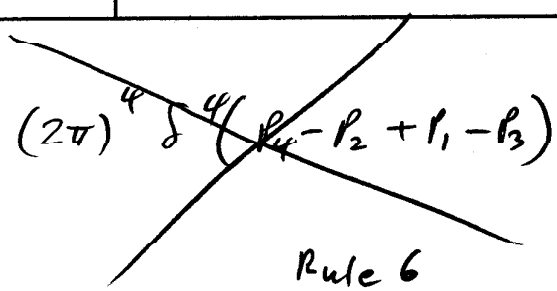
$$\times \underset{\text{Rule 4}}{(2\pi)^4 \delta^4(q + p_2 - p_4)} \; \underset{\text{Rule 5}}{\frac{d^4q}{(2\pi)^4}}$$

$$= (-i) \; g^2 \int \frac{(2\pi)^4 \delta^4(p_1 - p_3 - q)}{q^2 - m_c^2}$$

$$\times \delta(q + p_2 - p_4) \, d^4q$$

$q + p_2 - p_4 = 0 \Rightarrow q = p_4 - p_2$

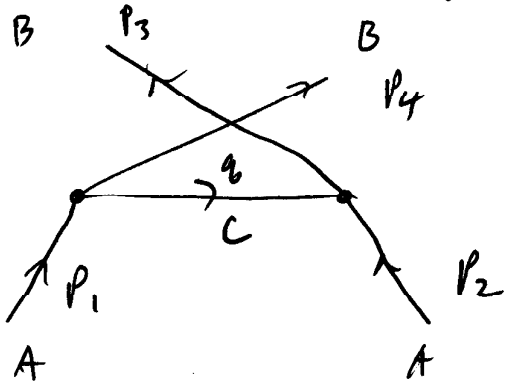
$$m = g^2 \frac{1}{(p_4 - p_2)^2 - m_c^2} (2\pi)^4 \delta^4(p_4 - p_2 + p_1 - p_3)$$



Rule 6

$$m_1 = \frac{g^2}{(p_4 - p_2)^2 - m_c^2}$$

Now we also have another diagram because m must be symmetric w.r.t. interchange of identical particles.



Note this diagram gives same result as first diagram if we replace p_4 with p_3:

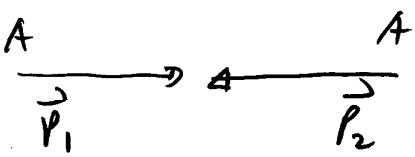
$$m_2 = \frac{g^2}{(p_3 - p_2)^2 - m_c^2}$$

Total $m = m_1 + m_2$

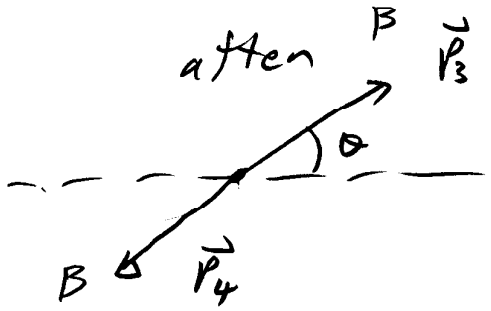
$$m = \frac{g^2}{(p_4 - p_2)^2 - m_c^2} + \frac{g^2}{(p_3 - p_2)^2 - m_c^2}$$

Lets calc. $\frac{d\sigma}{d\Omega}$ for this process in CM frame assuming $m_A = m_B \equiv m$ and $m_c = 0$.

before



after



$E_1 = E_2 \equiv E$, $E_3 = E_4 = E$

$E^2 = |\vec{p}_1|^2 + m^2 = |\vec{p}_3|^2 + m^2 \Rightarrow |\vec{p}_1| = |\vec{p}_3| \equiv |\vec{p}|$

$\vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4 = 0$

$$(p_4 - p_2)^2 = p_4^2 + p_2^2 - 2 p_4 \cdot p_2$$

$$= m^2 + m^2 - 2\left(E_4 E_2 - \vec{p}_4 \cdot \vec{p}_2\right)$$

$$= 2m^2 - 2\left(E^2 - |\vec{p}|^2 \cos\theta\right)$$

$$= 2m^2 - 2\left(|\vec{p}|^2 + m^2 - |\vec{p}|^2 \cos\theta\right)$$

$$= -2|\vec{p}|^2\left(1 - \cos\theta\right)$$

$$(p_3 - p_2)^2 = p_3^2 + p_2^2 - 2\, p_3 \cdot p_2$$

$$= m^2 + m^2 - 2\left(E^2 + |\vec{p}|^2 \cos\theta\right)$$

$$= -2|\vec{p}|^2 (1 + \cos\theta)$$

$$\therefore \quad m = \frac{g^2}{-2|\vec{p}|^2(1-\cos\theta)} + \frac{g^2}{-2|\vec{p}|^2(1+\cos\theta)}$$

$$= \frac{g^2(1+\cos\theta) + g^2(1-\cos\theta)}{-2|\vec{p}|^2(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{2g^2}{-2|\vec{p}|^2(1-\cos^2\theta)} = \frac{-g^2}{|\vec{p}|^2 \sin^2\theta}$$

From Eq. 6.42 :

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S\,|m|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|} \qquad S = \frac{1}{2}\,!\,\frac{1}{2}$$

$$= \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\left(\frac{1}{2}\right)|m|^2}{(2E)^2}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{q^4}{|\vec{p}|^4 \sin^4\theta} \frac{1}{8E^2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{\hbar c\, q^2}{16\pi E |\vec{p}|^2 \sin^2\theta}\right)^2}$$