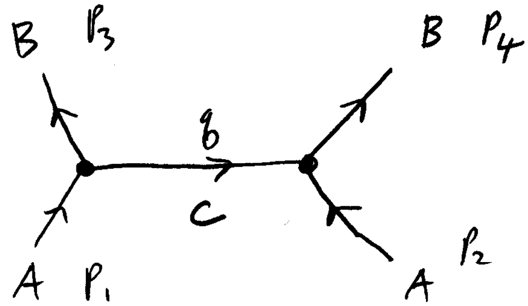
Sec. 6.5

consider scattering process

$$A + A \to B + B$$

Lowest order diagrams include



Rule 1
Label diagram

Apply rules:

$$-iM = \int \underbrace{(-ig)}_{\text{Rule 2}} \underbrace{\frac{i}{q^2 - m_c^2}}_{\text{Rule 3}} \underbrace{(-ig)}_{\text{Rule 2}} \underbrace{(2\pi)^4 \delta^4(p_1 - p_3 - q)}_{\text{Rule 4}}$$

$$\times \underbrace{\cancel{(2\pi)^4} \delta^4(q + p_2 - p_4)}_{\text{Rule 4}} \underbrace{\frac{d^4 q}{\cancel{(2\pi)^4}}}_{\text{Rule 5}}$$

$$= (-i) \quad g^2 \int \frac{(2\pi)^4 \delta^4(p_1 - p_3 - q)}{q^2 - m_c^2}$$

$$\times \quad \delta(q + p_2 - p_4) \; d^4 q$$

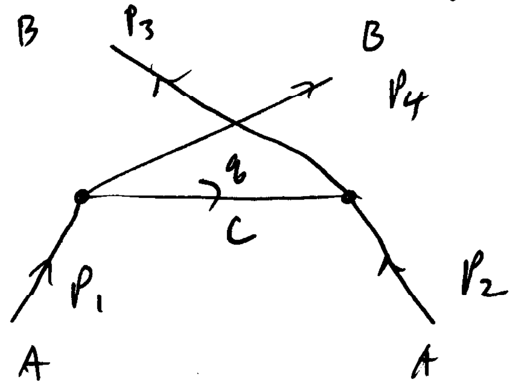
$$q + p_2 - p_4 = 0 \Rightarrow q = p_4 - p_2$$

$$\mathcal{M} = g^2 \frac{1}{(p_4-p_2)^2 - m_c^2} \cancel{(2\pi)^4 \delta^4(p_4 - p_2 + p_1 - p_3)}$$

Rule 6

$$\boxed{\mathcal{M}_1 = \frac{g^2}{(p_4-p_2)^2 - m_c^2}}$$

Now we also have another diagram because $\mathcal{M}$ must be symmetric w.r.t. interchange of identical particles.



Note this diagram gives same result as first diagram if we replace p_4 with p_3:

$$\boxed{\mathcal{M}_2 = \frac{g^2}{(p_3-p_2)^2 - m_c^2}}$$

Total $\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$

$$\mathcal{M} = \frac{g^2}{(p_4 - p_2)^2 - m_c^2} + \frac{g^2}{(p_3 - p_2)^2 - m_c^2}$$

Lets calc. $\frac{d\sigma}{d\Omega}$ for this process in CM frame assuming $m_A = m_B \equiv m$ and $m_c = 0$.

$E_1 = E_2 \equiv E$, $E_3 = E_4 = E$

$E^2 = |\vec{p}_1|^2 + m^2 = |\vec{p}_3|^2 + m^2 \Rightarrow |\vec{p}_1| = |\vec{p}_3| \equiv |\vec{p}|$

$\vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4 = 0$

$$\begin{aligned}
(p_4 - p_2)^2 &= p_4^2 + p_2^2 - 2 p_4 \cdot p_2 \\
&= m^2 + m^2 - 2\left(E_4 E_2 - \vec{p}_4 \cdot \vec{p}_2\right) \\
&= 2m^2 - 2\left(E^2 - |\vec{p}|^2 \cos\theta\right) \\
&= \cancel{2m^2} - 2\left(|\vec{p}|^2 + \cancel{m^2} - |\vec{p}|^2 \cos\theta\right) \\
&= -2|\vec{p}|^2 (1 - \cos\theta)
\end{aligned}$$

$$(p_3-p_2)^2 = p_3^2 + p_2^2 - 2\, p_3\cdot p_2$$

$$= m^2 + m^2 - 2\left(E^2 + |\vec{p}|^2 \cos\theta\right)$$

$$= -2|\vec{p}|^2 (1+\cos\theta)$$

$$\therefore\ \mathcal{M} = \frac{g^2}{-2|\vec{p}|^2(1-\cos\theta)} + \frac{g^2}{-2|\vec{p}|^2(1+\cos\theta)}$$

$$= \frac{g^2(1+\cancel{\cos\theta}) + g^2(1-\cancel{\cos\theta})}{-2|\vec{p}|^2(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{2g^2}{-2|\vec{p}|^2(1-\cos^2\theta)} = \frac{-g^2}{|\vec{p}|^2\sin^2\theta}$$

From Eq. 6.42:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2}\frac{|\vec{p}_f|}{|\vec{p}_i|} \qquad S = \frac{1}{2!} = \frac{1}{2}$$

$$= \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\left(\frac{1}{2}\right)|\mathcal{M}|^2}{(2E)^2}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g^4}{|\vec{p}|^4 \sin^4\theta} \frac{1}{8E^2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{\hbar c\, g^2}{16\pi E |\vec{p}|^2 \sin^2\theta}\right)^2}$$