Sec. 6.2 (con't) Elom

and the dirac delta ' s effect causes

$$\boxed{\Gamma = \frac{S * |M|^2 * \rho 0}{2 * (4\pi) * \hbar * m1^2 * c},}$$

$$\text{where } \rho 0 = \frac{c}{2 * m1} * \sqrt{m1^4 + m2^4 + m3^4 - 2\, m1^2\, m2^2 - 2\, m1^2\, m3^2 - 2\, m2^2\, m2^2}$$

Value of $|\vec{p}_2|$ when $E = m_1 c^2$

■ ~~Cross-Section~~ Scattering Golden Rule

$$1 + 2 \to 3 + 4 + \ldots + n$$

$$d\sigma = \left| M \right|^2 * \frac{S * \hbar^2}{4\sqrt{(p1 \bullet p2)^2 - (m1\, m2\, c^2)^2}} * \left[\frac{c\, d^3\, p3}{(2*\pi)^3 * 2 * E3} \times \ldots \times \frac{c\, d^3\, pn}{(2*\pi)^3 * 2 * En} \right] * (2*\pi)^4 * \delta^4\, (p1 + p2 - p3 - p4 \ldots - pn)$$

- $d\sigma$ gives cross section where **p3** is in range d^3 **p3**, p4 is in range d^3 **p4**, etc.

  Typically look for angle at which particle 3 emerges. So integrate over all other final momenta and |p3|

  Find differential cross section $d\sigma/d\Omega$ for scattering of particle 3 in solid angle $d\Omega$

- pi is 4-momentum pi=(Ei/c, **pi**)
  $Ei^2 = |\,pi\,|^2 + (mi * c)^2$

  δ enforces conservation of E and p: p1+p2 = p3+...+pn

  $S = \prod_{j=1}^{n} \left(\frac{1}{j!}\right)$ for j multiplicity of each particle.

  Generally M is a function of p2, p3,.., pn so it stays in integrand.

- **Example of 2 body scattering in CM frame**

- 1+2→3+4

In CM: **p2 = -p1**
m2, m3 ≠ 0
M (p2, p3)

$p1 = (E1/c, \mathbf{p1})$
$p2 = (E2/c, \mathbf{p2}) = (E2/c, -\mathbf{p1})$
$p3 = (E3/c, \mathbf{p3}) = \left(\sqrt{\mathbf{p3}^2 + (m3 * c)^2}, \mathbf{p3}\right)$
$p4 = (E4/c, \mathbf{p4}) = \left(\sqrt{\mathbf{p4}^2 + (m4 * c)^2}, \mathbf{p4}\right)$

Observe that $p1 \cdot p2 = E1 * E2/c^2 + p1^2$ which implies that $\sqrt{(p1 \cdot p2)^2 + (m1\, m2\, c^2)^2} = (E1 + E2) * |\mathbf{p1}|/c$

So

$$d\sigma = \left(\frac{\hbar * c}{8\pi}\right)^2 * \frac{S * |M|^2 * c}{(E1+E2) * |\mathbf{p1}|} \frac{1}{E3 * E4} * \delta^4(p1 + p2 - p3 - p4)\, d^3\mathbf{p2}\, d^3\mathbf{p3}$$

$$\delta^4(p1 + p2 - p3 - p4) = \delta\left(\frac{E1+E2-E3-E4}{c}\right) * \delta^3(-\mathbf{p3} - \mathbf{p4})$$

Replacing δ^4 by new form and evaluating $d^3\,\mathbf{p3}$ where we substitue **p4=-p3**

$$d\sigma = \left(\frac{\hbar}{8\pi}\right)^2 * \frac{S * |M|^2 * c}{(E1+E2) * |\mathbf{p1}|} * \frac{*\delta\left(\frac{E1+E2}{c} - \sqrt{\mathbf{p3}^2 + (m4 * c)^2} - \sqrt{\mathbf{p3}^2 + (m3 * c)^2}\right)}{\sqrt{\mathbf{p3}^2 + (m3 * c)^2} * \sqrt{\mathbf{p3}^2 + (m4 * c)^2}}\, d^3\mathbf{p3}$$

In this case $|M|^2$ depends on direction of p3 (angles) as well as magnitude so cannot integrate over angles as before.

$$d^3\mathbf{p3} = |\mathbf{p3}|^2 * d\,|\mathbf{p3}| * d\Omega$$

So

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 * \frac{S * c}{(E1+E2) * |\mathbf{p1}|} \int \frac{|M|^2 * \delta\left(\frac{E1+E2}{c} - \sqrt{\mathbf{p3}^2 + (m4 * c)^2} - \sqrt{\mathbf{p3}^2 + (m3 * c)^2}\right)}{\sqrt{\mathbf{p3}^2 + (m3 * c)^2} * \sqrt{\mathbf{p3}^2 + (m4 * c)^2}} * \left|\mathbf{p3}\right|^2 * d^3\mathbf{p3}$$

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So

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 * \frac{S * c}{(E1 + E2) * |p1|}$$
$$\int \frac{|M|^2 * \delta\left(\frac{E1+E2}{c} - \sqrt{\mathbf{p3}^2 + (m4 * c)^2} - \sqrt{\mathbf{p3}^2 + (m3 * c)^2}\right)}{\sqrt{\mathbf{p3}^2 + (m3 * c)^2} * \sqrt{\mathbf{p3}^2 + (m4 * c)^2}} * \left|\mathbf{p3}\right|^2 * d^3\mathbf{p3}$$

Let $\rho = |\,p3\,|$ and $E = c * \left(\sqrt{\mathbf{p3}^2 + (m3 * c)^2} + \sqrt{\mathbf{p3}^2 + (m4 * c)^2}\right)$

$$\frac{dE}{d\rho} = \frac{E * \rho}{\sqrt{\mathbf{p3}^2 + (m3 * c)^2} * \sqrt{\mathbf{p3}^2 + (m4 * c)^2}}$$

Hence $\frac{d\sigma}{d\Omega} = \left(\frac{\hbar * c}{8\,\pi}\right)^2 * \frac{S * |\,M\,|^2}{(E1 + E2)^2} * \frac{|\,\mathbf{p3}\,|}{|\,\mathbf{p1}\,|}$

Units

Lifetime → seconds

Decay rates → 1/seconds

Cross sections → Area 1 barn = 10^{-24} cm^2

M → depends on # of particles

Dim of M = $(mc)^{4-n}$