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CHAPTER 6: THE FEYNMAN CALCULUS

6.1: LIFETIMES AND CROSSSECTIONS

Average lifetime (τ)

The predictions of decay can be stated in terms of half life, decay constant or the average lifetime. "Life time" is simply the time takes to a certain particle to decay, but it is Impossible to calculate the lifetime of any particular particle. So instead we introduce average or mean lifetime, τ, of particles in any large sample.

Decay Rate (Γ)

"Decay rate" is number of decays per unit time.

$$\Gamma = Decay\ Rate = \frac{\#\ of\ decays}{time} \tag{1}$$

Consider a large sample of particles $N(t)$, at time t, and dN number of particles would decay in time dt, if the decay rate is Γ, the no of particles decay in time interval dt can be expressed as,

$$dN = -\Gamma N\, dt$$

$$\Rightarrow \int_{N(0)}^{N(t)} \frac{dN}{N} = -\int_0^t \Gamma\, dt$$

$$\boxed{N(t) = N(0)\, e^{-\Gamma t}} \tag{2}$$

Number of particles in the left, decreases exponentially with time.

The probability for decay can be expressed as a distribution function

$$f_{decay}(t) = A\, N(0)\, e^{-\Gamma t}$$

Where, A is the normalization constant, and it can be found as,

$$\int_0^{\infty} A\, N(0)\, e^{-\Gamma t}\, dt = 1 \Rightarrow A = \frac{\Gamma}{N(0)}$$

The distribution function now becomes,

$$f_{decay}(t) = \Gamma e^{-\Gamma t}$$

The expectation value of the distribution will result the average lifetime and it is given by,

$$< t > = \tau = \frac{\int_0^\infty t f_{decay}\, dt}{\int_0^\infty f_{decay}\, dt} = \int_0^\infty \Gamma t\, e^{-\Gamma t} dt = \frac{1}{\Gamma}$$

$$\boxed{\tau = \frac{1}{\Gamma}} \qquad (3)$$

When particles decay through several different routes, the total decay rate Γ_{Tot}, is the sum of the individual decay rates.

$$\Gamma_{Tot} = \sum_{i=1}^{n} \Gamma_i \qquad where \qquad \tau = \frac{1}{\Gamma_{Tot}}$$

Branching Ratio

Branching ratio is the fraction of all particles of the given type that decay by each mode.

$$\boxed{Branching\ Ratio\ for\ i\ th\ decay\ \mathrm{mod}e = \frac{\Gamma_i}{\Gamma_{Tot}}} \qquad (4)$$

Cross Sections

Cross section can be defined as the effective surface area of the impact for a given collision.

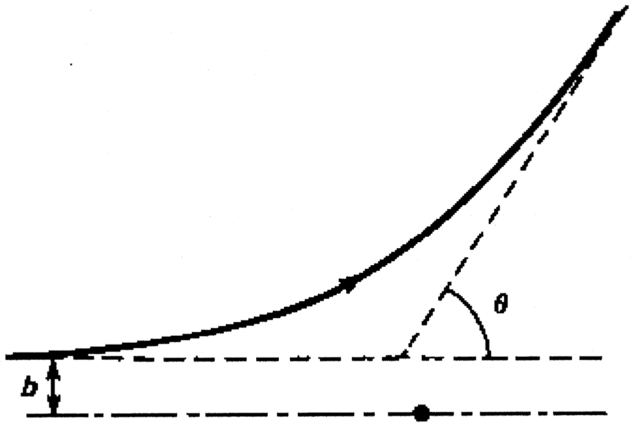

Fig 6.1 Scattering from a fixed potential: θ is the scattering angle and b is the impact parameter.

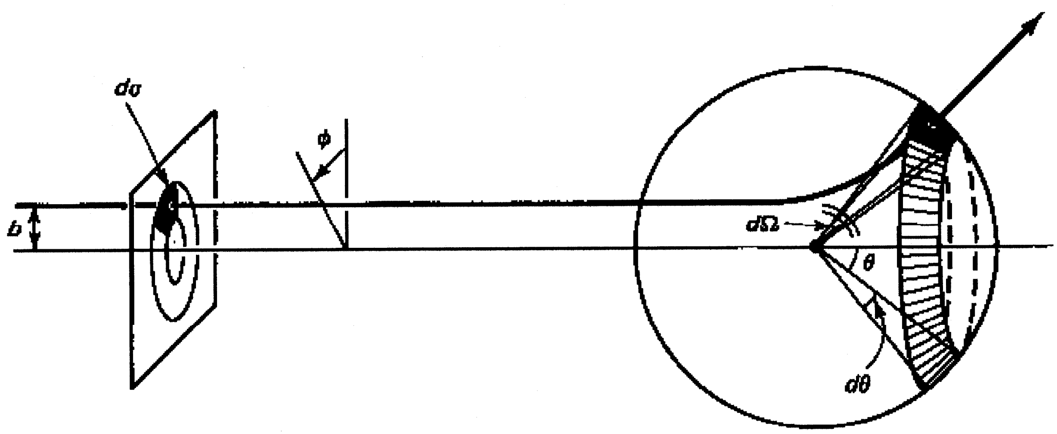

Fig 6.2 : Particle incident in area $d\sigma$ scatters in to solid angle $d\Omega$

Differential Cross section $D(\theta)$

$$D(\theta) = \frac{d\sigma}{d\Omega} = \frac{\text{Number of particles scattered into solid angle d}\Omega\text{ per unit time}}{\text{Incident intensity}}$$

From the figure,

$d\sigma = |b\ db\ d\phi|$ *and* $d\Omega = |\sin\theta\ d\theta\ d\phi|$

$$\Rightarrow \boxed{D(\theta) = \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left(\frac{db}{d\theta}\right)} \qquad (5)$$

Total cross section σ

$$\sigma = 2\pi \int_0^{\pi} \left(\frac{d\sigma}{d\Omega}\right) \sin\theta\ d\theta \qquad \text{with } d\Omega = 2\pi \sin\theta\ d\theta$$

Units : barn 1Barn = 10^{-24} cm^2

Example 6.1

Hard sphere Scattering

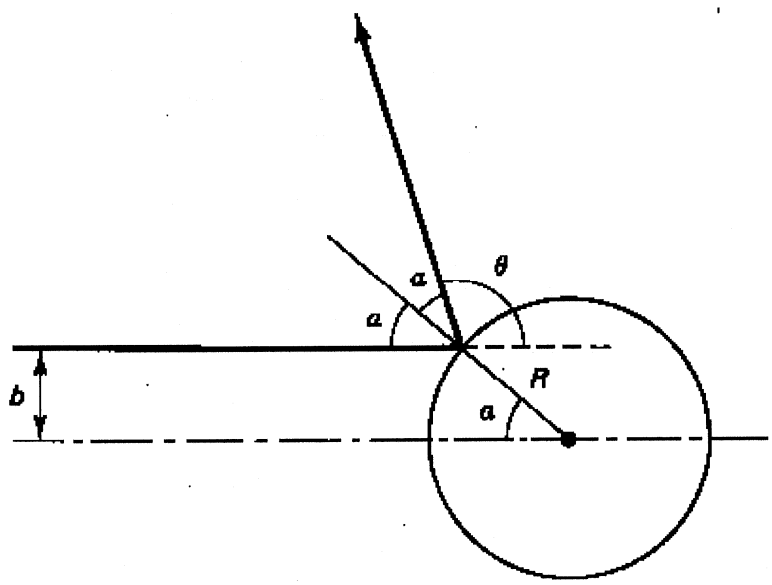

Fig 6.3: Hard sphere scattering

Consider that the particle bounces elastically off a sphere of radius R .

From figure 6.3
$2\alpha + \theta = \pi$

$$Sin\alpha = Sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = Cos\frac{\theta}{2}$$

$$b = R\ Sin\alpha \quad \Rightarrow \quad b = R\ Cos\frac{\theta}{2} \qquad\qquad \theta = 2\ Cos^{-1}\left(\frac{b}{R}\right)$$

$$\frac{db}{d\theta} = -\frac{R}{2}\,Sin\frac{\theta}{2}$$

$$D(\theta) = \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left(\frac{db}{d\theta}\right) = \left|\frac{-bR\ Sin\frac{\theta}{2}}{2\ Sin\theta}\right|$$

$$\boxed{D(\theta) = \frac{R^2}{4}}$$

$$Total\ Cross\ section\ \sigma = \int d\sigma = \int D(\theta)\ d\Omega$$

$$= \int \frac{R^2}{4} Sin\ \theta\ d\theta\ d\phi$$

$$\boxed{\sigma = \pi\ R^2}$$

Example 6.2

Rutherford scattering

A particle of charge q_1 scatters off a stationary particle of charge q_2.

The impact parameter for this scattering can be derived as,

$b = \frac{q_1 q_2}{2E}\ Cot\frac{\theta}{2}$ where E is the kinetic energy of the incident charge.

The differential cross section can be calculated using equation (5),

$$\boxed{D(\theta) = \left(\frac{q_1 q_2}{4E\ Sin^2\frac{\theta}{2}}\right)^2}$$

The total cross section is

$$\sigma = 2\pi \left(\frac{q_1 q_2}{4E} \right)^2 \int_0^{\pi} \frac{Sin\,\theta}{Sin^2 \frac{\theta}{2}} \, d\theta \quad \Rightarrow \quad \boxed{\sigma = \infty}$$

Differential cross section in terms of Luminosity

Luminosity, L , is the number of particles passing down a line per unit time per unit area.

Consider a beam of incoming particles with uniform Luminosity L .

The no. of particles per unit time passing through area $d\sigma$, or no. of particles per unit time scattered in to solid angle $d\Omega$ can be define as,

$$dN = L d\sigma = L\, D(\theta)\, d\Omega$$

It follows,

$$\boxed{D(\theta) = \frac{d\sigma}{d\Omega} = L \frac{dN}{d\Omega}}$$