CP violation in SM

Yang Presentation

I: C. P transformation on various quantities

	$\bar\psi_i\psi_j$	$i\bar\psi_i\gamma^5\psi_j$	$\bar\psi_i\gamma^\mu\psi_j$	$\bar\psi_i\gamma^\mu\gamma^5\psi_j$	A^μ	$W^\mu_\pm$	∂_μ
P	$+$	$-$	$(-)^\mu$	$(-)(-)^\mu$	$(-)^\mu$	$(-)^\mu$	$(-)^\mu$
C	$\bar\psi_j\psi_i$	$i\bar\psi_j\gamma^5\psi_i$	$-\bar\psi_j\gamma^\mu\psi_i$	$\bar\psi_j\gamma^\mu\gamma^5\psi_i$	$-$	$-W^\mu_\mp$	$+$
CP	$\bar\psi_j\psi_i$	$-i\bar\psi_j\gamma^5\psi_i$	$(-)(-)^\mu\bar\psi_j\gamma^\mu\psi_i$	$(-)(-)^\mu\bar\psi_j\gamma^\mu\gamma^5\psi_i$	$(-)(-)^\mu$	$(-)(-)^\mu W^\mu_\mp$	$(-)^\mu$

II C.P conserve seperately in QED

$$\mathcal{L} = \mathcal{L}_0 - e\bar\psi\gamma^\mu\psi A_\mu$$

$$= \bar\psi(i\not\partial - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar\psi\gamma^\mu\psi A_\mu$$

$$= \bar\psi i\gamma^\mu\partial_\mu\psi - m\bar\psi\psi - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - e\bar\psi\gamma^\mu\psi A_\mu$$

P: $(-)(-)$ $\quad +$ $\quad +$ $\quad (-)^\mu(-)^\mu$ $\quad (-)(-)$

C: $(-)\partial_\mu\bar\psi i\gamma^\mu\psi$ $\quad +$ $\quad +$

$$\llcorner\ (-)\{\partial_\mu(\bar\psi i\gamma^\mu\psi) - \bar\psi i\gamma^\mu\partial_\mu\psi\}$$

$$= \bar\psi i\gamma^\mu\partial_\mu\psi$$

III: C, P violate seperately in V-A theory, but the combined transformation conserves in lepton section.

$$\mathcal{L}_I = -\frac{g}{\sqrt{2}}\left\{\bar{\psi}_{\nu_l}\gamma^\mu\frac{1-\gamma^5}{2}\psi_l W^+_\mu + \bar{\psi}_l\gamma^\mu\frac{1-\gamma^5}{2}\psi_{\nu_l}W^-_\mu\right\}$$

$$\propto\left\{\bar{\psi}_{\nu_l}\gamma^\mu\psi_l W^+_\mu + \bar{\psi}_l\gamma^\mu\psi_{\nu_l}W^-_\mu - \bar{\psi}_{\nu_l}\gamma^\mu\gamma^5\psi_l W^+_\mu - \bar{\psi}_l\gamma^\mu\gamma^5\psi_{\nu_l}W^-_\mu\right\}$$

P: $(-)^\mu\ (-)^\mu \qquad (-)^\mu\ (-)^\mu \qquad (-)(-)^\mu\ (-)^\mu \qquad (-)(-)^\mu\ (-)^\mu$

C: $(-)\bar{\psi}_l\gamma^\mu\psi_{\nu_l}(-)W^-_\mu + (-)\bar{\psi}_{\nu_l}\gamma^\mu\psi_l(-)W^+_\mu$

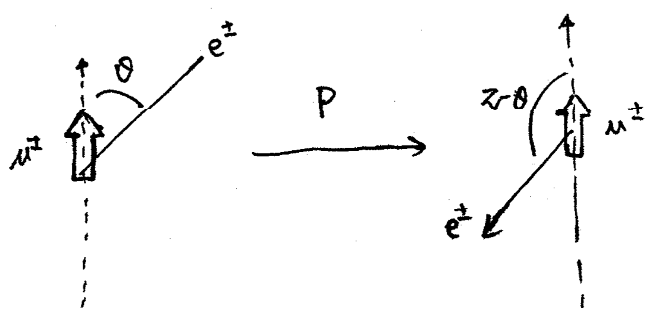
$-\bar{\psi}_l\gamma^\mu\gamma^5\psi_{\nu_l}(-)W^+_\mu - \bar{\psi}_{\nu_l}\gamma^\mu\gamma^5\psi_l(-)W^+_\mu$

CP: $\bar{\psi}_l\gamma^\mu\psi_{\nu_l}W^-_\mu + \bar{\psi}_{\nu_l}\gamma^\mu\psi_l W^+_\mu$

$-\bar{\psi}_l\gamma^\mu\gamma^5\psi_{\nu_l}W^+_\mu - \bar{\psi}_{\nu_l}\gamma^\mu\gamma^5\psi_l W^+_\mu$

Experiment: $\mu^\pm$ decay

$$\begin{cases}\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \\ \mu^+ \to e^+ + \nu_e + \bar{\nu}_e\end{cases}$$



The angular distribution of the electron (positrons) emitted in $\mu^\pm$ decay are given by.

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm\left(1-\frac{\xi_\pm}{3}\cos\theta\right)$$

$$\begin{cases} \xi_\pm \text{ — "asymmetry parameters"} \\ \Gamma_\pm \text{ — } \frac{1}{\tau_\pm} = \int_{-1}^{+1}\Gamma_\pm(\cos\theta)\,d\cos\theta \end{cases}$$

△ Charge conjugation

If invariant, then

$$\Gamma_{\mu^+}(\cos\theta) = \Gamma_{\mu^-}(\cos\theta) \Rightarrow \begin{cases} \Gamma_+ = \Gamma_- \\ \xi_+ = \xi_- \end{cases}$$

△ Parity

If invariant, then

$$\Gamma_{\mu^\pm}(\cos\theta) = \Gamma_{\mu^\pm}(-\cos\theta) \Rightarrow \xi_+ = \xi_- = 0$$

△ Experiment results.

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = -\xi_- = 1.00 \pm 0.04$$

So, both C and P are violated.

△ Combined operation. CP.

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm\left(1-\frac{\xi_\pm}{3}\cos\theta\right)$$

IV CP - nonconservation shows up as a rate different between two processes that are the CP conjugates of one - another.

$$A \rightarrow B \qquad \qquad CP: \bar{A} \rightarrow \bar{B}$$

$$\mathcal{M} = \mathcal{M}(A \rightarrow B) = g_1 r_1 e^{i\phi_1} + g_2 r_2 e^{i\phi_2}$$

g_1, g_2 : Coupling constant

$r_i e^{i\phi_i}$: Strong phase.

$$\mathcal{M}_{cp} = \mathcal{M}(\bar{A} \rightarrow \bar{B}) = g_1^* r_1 e^{i\phi_1} + g_2^* r_2 e^{i\phi_2}$$

$$|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2 = 2 r_1 r_2 \, \mathrm{Im}(g_1 g_2^*) \sin(\phi_1 - \phi_2)$$

$$= \frac{1}{2} \Gamma_{\mp} \left(1 - \frac{(-) \xi_{\mp} \cos\theta}{3} \right)$$

$$= \frac{1}{2} \Gamma_{\mp} \left[1 - \frac{\xi_{\mp}}{3} (-\cos\theta) \right]$$

$$= \Gamma_{a\mp} (\cos \pi - \theta)$$

IV cp violation from CKM matrix

① $$\mathcal{L}_I \propto J^{+\mu} W^+_\mu + J^{-\mu} W^-_\mu$$

$$\propto \overline{(u, c, t)_L} V \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L + \overline{(d, s, b)} V^\dagger \gamma^\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

② $$\begin{pmatrix} d \\ s \\ b \end{pmatrix}' = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V = R_2(-\theta_2) R_1(-\theta_1) D(\delta - \pi) R_2(\theta_3)$$

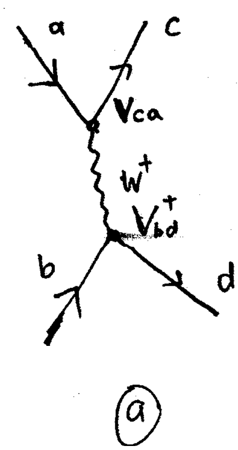
$$R_1(\theta_i) = \begin{pmatrix} c_i & s_i & 0 \\ -s_i & c_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R_2(\theta_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_i & s_i \\ 0 & -s_i & c_i \end{pmatrix}$$

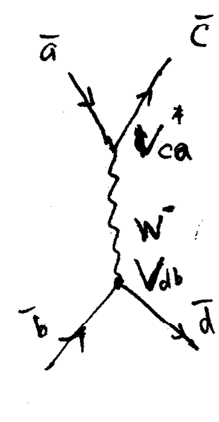
$$D(\delta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix}$$

③ Amplitude

a: $ab \to cd$

b: $\bar{a}\bar{b} \to \bar{c}\bar{d}$





$$\mathcal{M}_a \sim J^{\mu}_{ca} J^{+}_{\mu bd} \sim [\bar{u}_{cL}\gamma^{\mu} V_{ca} u_{aL}][\bar{u}_{dL}\gamma_{\mu} V^{+}_{bd} V_{bL}]$$

$$\sim V_{ca} V^{*}_{db} [\bar{u}_{cL}\gamma^{\mu} u_{aL}][\bar{u}_{dL}\gamma_{\mu} u_{bL}]$$

$$\mathcal{M}_b \sim V^{*}_{ca} V_{db} [\bar{u}_{aL}\gamma^{\mu} u_{cL}][\bar{u}_{bL}\gamma_{\mu} u_{dL}] = \mathcal{M}_a^{+}$$

$$\mathcal{M}_{cp} \sim (J^{\mu}_{ca})_{cp} (J^{\mu}_{\mu bd})^{+}_{cp}$$

$$\sim V_{ca} V^{*}_{db} [\bar{u}_{aL}\gamma^{\mu} u_{cL}][\bar{u}_{bL}\gamma_{\mu} u_{dL}]$$

If V matrix is real then

$\mathcal{M}_{cp} = \mathcal{M}_a^{+}$ CP invariant.

If V matrix is complex $e^{i\delta}$, then.

$\mathcal{M}_{cp} \neq \mathcal{M}_a^{+}$ CP violation