by
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Lifetimes and cross sections.

* There are three experiments to ~~b~~ explain particle interactions;

1): Bound States
2): Decays
3): Scattering

* Bound states are described using quantum mechanics non-relativistically (see chapter 5).

* Both decays and scattering are described relativistically by using Feynman's formulation.

By using Feynman's calculation, we can gather information about the interaction. Such as it's result and type.

In Scattering experiments such as Rutherford, and Yukawa, we can make predictions about the outcome of the experiment. These experiments are of great importance, as many particle interactions involve a central force such as a single nucleus, scattering a particle such as an α particle.

Decay reactions can give us the lifetime of a particle, for example a muon. This lifetime gives us the probability that a decay will take place.

In order to calculate the lifetime of a particle, we need to use the equation;

$$dN(t) = -\Gamma N(t)\, dt \qquad (1)$$

Where Γ is the probability per unit time that any particle with decay, and is known as the decay rate.

We now need to integrate this equation to give us a relationship between the number of particles initially (N_0) and the number of particles at a given time t, ($N(t)$).

$$\int_0^N \frac{dN(t)}{N(t)} = -\int_0^t \Gamma\, dt.$$

$$\ln\left(\frac{N(t)}{N_0}\right) = -\Gamma t.$$

$$\frac{N(t)}{N_0} = e^{-\Gamma t}$$

Therefore $N(t) = N_0 e^{-\Gamma t}$. (2).

We can now show that the decay rate, Γ is related to the lifetime, τ, by using a factor k.

$$\int_0^\infty kN(t)\, dt = 1.$$

Since $N(t) = N_0 e^{-\Gamma t}$, we get;

$$\int_0^\infty k N_0 e^{-\Gamma t}\, dt = 1$$

$$k N_0 \int_0^\infty e^{-\Gamma t}\, dt = 1.$$

$$k N_0 \left[-\frac{1}{\Gamma} \left(e^{-\infty} - e^{0} \right) \right] = 1$$

$$\frac{k N_0}{\Gamma} = 1, \text{ or } k = \frac{\Gamma}{N_0}$$

so now $\tau = \int_0^\infty k t N(t)\, dt$

$$= \int_0^\infty k t N_0 e^{-\Gamma t}\, dt$$

$$= k N_0 \int_0^\infty t e^{-\Gamma t}\, dt.$$

$$= \int_0^\infty t e^{-\Gamma t}\, dt$$

$$= \left[t \left(\frac{e^{-\Gamma t}}{-\Gamma} \right) \right]_0^\infty - \left[\left(\frac{e^{-\Gamma t}}{-\Gamma^2} \right) \right]_0^\infty$$

which then gives us; $k N_0 \int t e^{-\Gamma t}\, dt = \frac{k N_0}{\Gamma^2}$

Now we have $\tau = \frac{kN_0}{\Gamma^2}$ and $k = \frac{\Gamma}{N_0}$

$$\Rightarrow \tau = \frac{\cancel{k}\,\cancel{N_0}}{\Gamma^2} \cdot \frac{\cancel{\Gamma}}{\cancel{N_0}}$$

$$\Rightarrow \tau = \frac{1}{\Gamma^{\cancel{2}}}$$

In decay reactions there are several routes particles can use.

For example, π^+ can decay in four different way;

$\pi^+ \rightarrow \mu^+ + \nu_\mu$
$\pi^+ \rightarrow e^+ + \nu_e$
$\pi^+ \rightarrow \mu^+ + \nu_\mu + \gamma$
$\pi^+ \rightarrow e^+ + \nu_e + \pi^0$

So now we need to determine the <u>total</u> decay rate.

Summing over all the individual rates we get $\Gamma_{tot} = \sum_{i=1}^{n} \Gamma_i$ and the lifetime now becomes $\tau = \frac{1}{\Gamma_{tot}}$.

1

Branching Ratios.

This is defined as being the ratio between decay rates of individual decay modes and the total decay rate.

The equation is; $B_{r_i} = \frac{\Gamma_i}{\Gamma_{tot}}$. for the ith decay

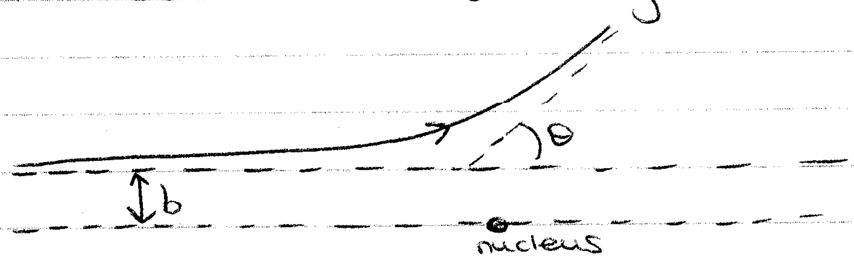
mode.

~~This now takes us into~~

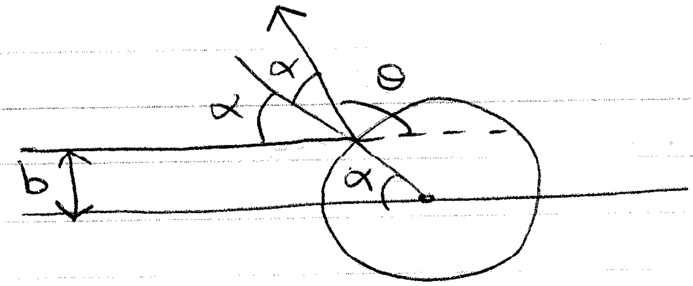
Scattering and cross sections

In a Scattering experiment, such as Rutherford, where an α particle is collided with a single gold nucleus, the parameter of importance to us is the cross sectional area during particle impact. i.e the size of the target.

In general a particle approaching the nucleus with an impact parameter of $b + db$ will be scattered at an angle of $\theta + d\theta$;



Imagine a particle that bounces elastically off a Sphere with radius R.



We have: $b = R\sin\alpha$, $2\alpha + \theta = \pi$

$$\sin\alpha = \sin(\pi/2 - \theta/2) = \cos(\theta/2)$$

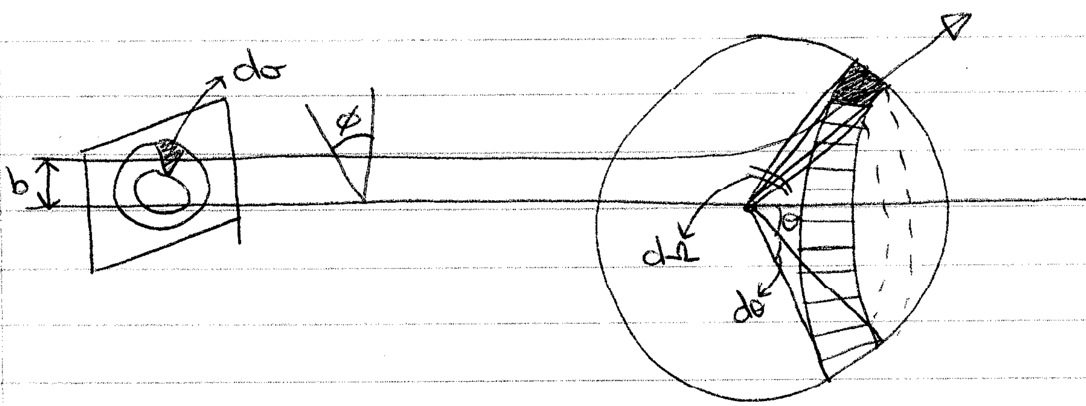
Putting into equations.

$$b = R\cos(\theta/2) \quad , \quad \theta = 2\cos^{-1}(b/R).$$

Since the nucleus is a sphere, the particle will be ~~Scatted~~ scattered by an angle, known as the solid angle, $d\Omega$. It passes through an area $d\sigma$, which is proportional to $d\Omega$, giving us the proportionality factor;

$$d\sigma = D(\theta)\, d\Omega .$$

This is also known as the differential scattering cross section, D.



If we want to calculate, D, we can use

$d\sigma = |b\, db\, d\phi|$ and $d\Omega = |\sin\theta\, d\theta\, d\phi|$

since $D(\theta)\, d\Omega = \dfrac{\text{number of scattered particles}}{\text{number of incident particles}}$

$$= d\sigma$$

$$D(\theta) = \frac{d\sigma}{d\Omega}$$

$$D(\theta) = \left| \frac{b\, db\, d\phi}{\sin\theta\, d\theta\, d\phi} \right| \Rightarrow D(\theta) = \left| \frac{b}{\sin\theta} \cdot \frac{db}{d\theta} \right|$$

In Rutherford Scattering, the scattering potential is described by Coulomb's Law;

$$F = \frac{q_1 q_2}{r^2}.$$

Since $F \propto \frac{1}{r^2}$

$$F(r) = -\frac{k}{r^2} \quad \text{where } k = q_1 q_2.$$

Therefore we can determine that the potential (at $k>0$), $V(r) = \int F(r)\,dr$

$$\Rightarrow V(r) = \frac{-k}{r}.$$

From this we can calculate the orbit of the particle around the nucleus.

$$\eta = \eta' - \int \frac{du}{\sqrt{\frac{2ME}{l^2} + \frac{2mku}{l^2} - u^2}}$$

~~$u = \frac{1}{r}$~~ $u = \frac{1}{r}$.

$$\cancel{\int \frac{dx}{\sqrt{\alpha + \beta x + \gamma x^2}} = \frac{1}{\sqrt{-\gamma}} \arccos\left(-\beta\right.}$$

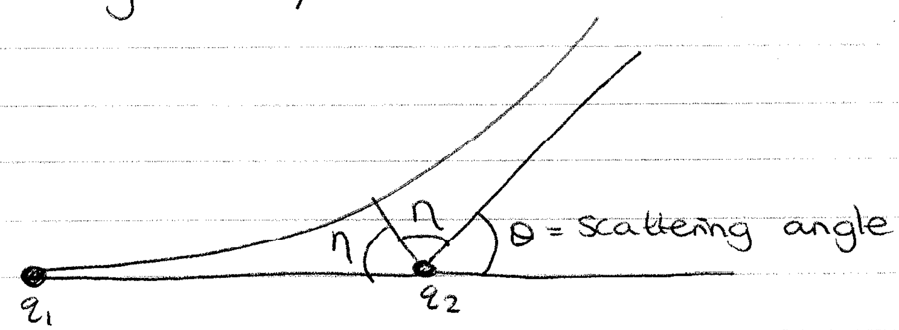
Derivation in Enrique's notes.

$$\frac{1}{r} = \frac{mk}{l^2}\left[1 + \sqrt{1 + \frac{2EL^2}{mk^2}}\cos(\eta - \eta')\right]$$

$\downarrow$ eccentricity of orbit.

Classically, the impact parameter, $b = \frac{q_1 q_2}{2E}\cot g(\theta/2)$.

To derive this, we need to look at the scattering model;



$k = q_1 q_2$.

$$\frac{1}{r}\ \frac{mq_1 q_2}{l^2}\left[1 + \sqrt{1 + \frac{2EL^2}{mk^2}}\cos(\eta - \eta')\right]$$

$$= \frac{1}{r}\ \frac{mq_1 q_2}{l^2}\left[1 + \varepsilon\cos(\eta - \eta')\right]$$

Since our closest approach parameters are;
$\eta' = \pi$ and $\eta = 0$.

we have $\frac{1}{r} = \frac{mq_1 q_2}{l^2}(\varepsilon\cos\eta - 1)$

as the orbit is determined by $r \to \infty$,

$$\frac{1}{\infty} = \frac{m q_1 q_2}{l^2}(\varepsilon \cos\eta - 1). \qquad = \varepsilon\cos\eta - 1.$$
$$= \cos\eta = \frac{1}{\varepsilon}.$$

$\theta + 2\eta = \pi$. so we have $\eta = \frac{\theta - \pi}{2}$

$\sin\left(\frac{\theta}{2}\right) = \frac{1}{\varepsilon} \qquad \varepsilon = \mathrm{cosec}\left(\frac{\theta}{2}\right)$

$\mathrm{cotg}^2\left(\frac{\theta}{2}\right) = \mathrm{cosec}^2\left(\frac{\theta}{2}\right) - 1$

So we then have $\mathrm{cotg}^2\left(\frac{\theta}{2}\right) = \varepsilon^2 - 1$.

Substitute ε

and we get the equation for the impact parameter, $b = \frac{q_1 q_2}{2E}\,\mathrm{cotg}(\theta/2)$

From this we can get the equation for $D(\theta)$;

$$D(\theta) = \left(\frac{q_1 q_2}{4E\sin^2(\theta/2)}\right)^2$$

To calculate the total cross section we need to use;

$d\sigma = D(\theta)\,d\Omega$.

$$\sigma = \int^{\pi} D(\theta)\,d\Omega \quad = (q_1 q_2)^2 \int^{2\pi} d\alpha \int^{\pi} \sin\theta\,d\theta$$

which will give us $\sigma = 2\pi \left(\frac{q_1 q_2}{4E}\right)^2 \int_0^\pi \frac{\sin\theta \, d\theta}{\sin^4(\theta/2)}$

This result will give us infinity, meaning that the coulomb potential has an infinite range. If we take a beam of particles with a uniform luminosity L, we can determine the number of particles per time passing through our are $d\sigma$.

$dN = L d\sigma$

$\quad = L D(\theta) d\Omega$

$$\frac{d\sigma}{d\Omega} = D(\theta) = \frac{1}{L} \frac{dN}{d\Omega}.$$

This gives us the differential cross section:

Can also be written in relation to the S matrix,

reduced masses and momenta of colliding system $= m_i \vec{p}_i$ and $m_f \vec{p}_f$.

$$\frac{d\sigma}{d\Omega} = (2\pi)^4 m_i m_f \frac{p_f}{p_i} |T_{fi}|^2$$

on-shell T-matrix;

$$S_{fi} = \delta_{fi} - 2\pi i \delta(E_f - E_i) \delta(\vec{p}_i - \vec{p}_f) T_{fi}$$

dirac delta function