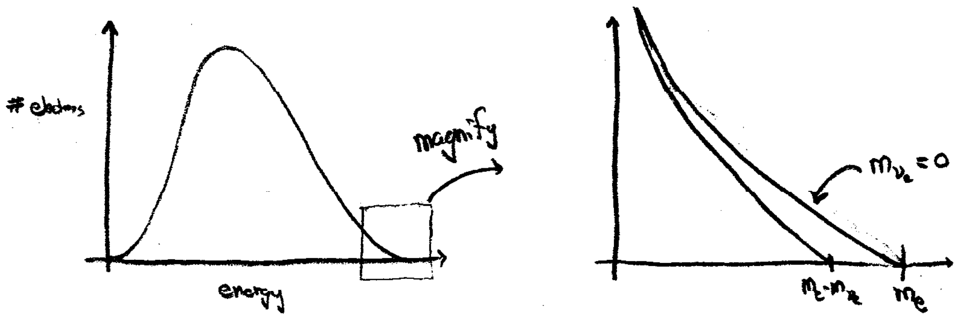
Neutrinos in the Standard Model

In the Standard Model, neutrinos are assumed to be massless. This is based both on evidence from experiment and theory. Experimentally, neutrinos have never been seen to have mass from some sort of direct measurement. Such a measurement might be measuring the endpoint of β-decay. Remember, in β-decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

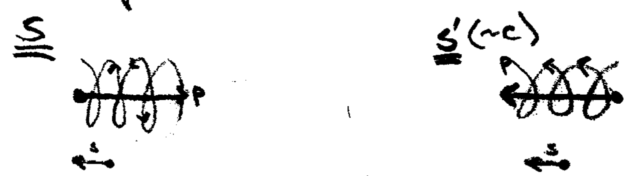
So, taking the energy spectrum of the electrons will give something like



It was because of the continuous energies of electrons in β-decay that Wolfgang Pauli first proposed the existence of neutrinos. Now, if we look at the endpoint of the energy spectrum, we would expect to see differences in the electron energies if the neutrinos have mass, because some of the energy in the decay must go into the mass of the neutrino. Current experimental limits put

$$m_{\nu_e} < 2.5 \text{ eV}/c^2$$

The theoretical reasons for assuming neutrinos are massless also is based on results obtained from β decay. We studied the ^{60}Co β-decay earlier that showed the weak force maximally violated parity: that is the antineutrinos produced were <u>always</u> right-handed. Also, it's been shown that neutrinos are left-handed. So, the spin and momentum are aligned for antineutrinos, and anti-aligned for neutrinos. So, consider the following picture



Say we observe a neutrino in frame S (note $\vec{p}$ and $\vec{s}$ anti-aligned). Then, it may be possible to boost to a frame S' that is faster than the neutrino. The $\vec{p}$ switches direction, but $\vec{s}$ does not. So, we would be able to see a right-handed neutrino. Since we never see right-handed neutrinos, it must be impossible to boost into such a frame where that is possible. So, it makes sense then that neutrinos are massless and so travel at the speed of light.

So, in the Standard Model, neutrinos are assumed to be massless.

Neutrino Oscillations and Atmospheric Neutrinos

So, let's assume for a moment that the neutrinos do have mass. Then we can describe neutrinos in one of two ways: their flavor eigenstates, ν_α, or their mass eigenstates, ν_i. Neutrinos are born and die in a particular flavor eigenstate; the weak force interacts with ν_e, ν_μ, and ν_τ. However, neutrinos propogate through space in particular mass eigenstates. To see this, let's consider an example with two neutrinos (we can generalize to three neutrinos later). So, we can write

$$\begin{pmatrix}\nu_e \\ \nu_x\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix} = \begin{pmatrix}\nu_1\cos\theta + \nu_2\sin\theta \\ -\nu_1\sin\theta + \nu_2\cos\theta\end{pmatrix}$$

Note that for $\theta = 0$, the two sets of eigenstates are precisely the same ($\nu_e = \nu_1$, $\nu_\mu = \nu_2$). Now the mass eigenstates, as a function of time, are

$$|\nu_i(t)\rangle = e^{-iE_it}|\nu_i(0)\rangle$$

if we say $\hbar = c = 1$. So, we can write

$$|\nu_e(t)\rangle = \cos\theta\, e^{-iE_1t}|\nu_1(0)\rangle + \sin\theta\, e^{-iE_2t}|\nu_2(0)\rangle$$

and

$$|\nu_e(0)\rangle = \cos\theta\,|\nu_1(0)\rangle + \sin\theta\,|\nu_2(0)\rangle$$

So, the probability of finding a neutrino, originally ν_e, in the ν_e state after time t is

$$|\langle\nu_e(t)|\nu_e(0)\rangle|^2$$

$$= |\cos^2\theta\, e^{iE_1t} + \sin^2\theta\, e^{iE_2t}|^2$$

$$= (\cos^2\theta\, e^{-iE_1t} + \sin^2\theta\, e^{-iE_2t})(\cos^2\theta\, e^{iE_1t} + \sin^2\theta\, e^{iE_2t})$$

$$= \cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta\left(e^{it(E_1-E_2)} + e^{it(E_2-E_1)}\right)$$

$$= \cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta\left(\cos t(E_1-E_2) + \cos t(E_2-E_1) + i\left(\sin t(E_1-E_2) + \sin t(E_2-E_1)\right)\right)$$

But since cos is even and sin is odd

$$= \cos^2\theta\sin^2\theta\,(2\cos t(E_2-E_1)) + \cos^4\theta + \sin^4\theta$$

So, recall that

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2\cos\theta\sin\theta$$

$$\cos^4\theta + \sin^4\theta = 1 - 2\cos^2\theta\sin^2\theta = 1 - \frac{\sin^2 2\theta}{2}$$

Therefore, we get now

$$P(\nu_e\to\nu_e) = \left(\tfrac{1}{4}\sin^2 2\theta\right)\left[2\left(1 - 2\sin^2\left(\tfrac{(E_2-E_1)t}{2}\right)\right)\right] + \left(1 - \tfrac{1}{2}\sin^2 2\theta\right)$$

$$= 1 - \sin^2 2\theta\,\sin^2\left[(E_2-E_1)t/2\right]$$

And so

$$P(\nu_e \to \nu_x) = 1 - P(\nu_e \to \nu_e) = \sin^2 2\theta \sin^2\left[(E_2 - E_1)\frac{t}{2}\right]$$

Now, let's assume for simplicity that both ν_1 and ν_2 have the same momentum, p. So then

$$E_i = \sqrt{m_i^2 + p^2} = p\sqrt{1 + \frac{m_i^2}{p^2}}$$

Now, if $p \gg m$, which is a good assumption for neutrinos (which we know are light) then the binomial expansion gives

$$E_i \approx p + \frac{m_i^2}{2p}$$

So, if we say $p \sim E$ (note, again we use $m \ll p$), then

$$E_2 - E_1 = \frac{m_2^2 - m_1^2}{2E} = \frac{\Delta m^2}{2E}$$

Now, if we want the probability in terms of length, approximate

$$t \sim L$$

Then

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(1.27\, \Delta m^2 \frac{L}{E}\right)$$

$$P(\nu_e \to \nu_x) = \sin^2 2\theta \sin^2\left(1.27\, \Delta m^2 \frac{L}{E}\right)$$

where 1.27 is just a constant to get proper units (L in km, Δm^2 in $(eV/c^2)^2$, and E in GeV). So, here

θ : mixing angle

Δm^2 : mass difference between ν_2 and ν_1

L : distance neutrino travels

E : energy of neutrinos

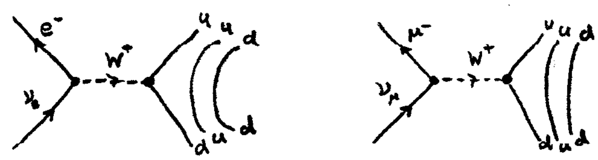
So, if Δm^2 is small, in order to see neutrino oscillations you need to look at large traveling distances.

Now, it is easy enough to generalize this to more than two neutrinos

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$$

where $U_{\alpha i}$ is the mixing matrix. While the whole idea of neutrinos oscillating between flavor states may seem strange, we have seen this before in the CKM matrix that described mixing of the down, strange, and bottom quarks. Also, the two neutrino case we derived above is similar to the K_0, $\bar{K}_0$ mixing we saw earlier.

To see evidence of neutrino oscillations, we can look at the resuts from the Super-Kamiokande, underground detector, capable of detecting Cerenkov radiation from electrons and muons produced in the charged current reactions



The production of highly energetic atmospheric neutrinos is isotropic, so we would expect the upward-going neutrino flux to equal the downward going neutrinos flux. As it turns out that is not the case: the downward going muon neutrino flux exceeds the upword going flux. This is because neutrinos travelling up must go a greater distance (through the earth) to reach the detector, and so they have more time to oscillate to a different flavor. There is no observed difference in the electron neutrino flux with zenith angle, so it is assumed that what is seen is $\nu_\mu \to \nu_\tau$.

SHOW FIGURE:

Because those results are integrated over a variety of energies and path lengths, we would expect the mean value

$$\sin^2\left(1.27\,\Delta m^2 \frac{L}{E}\right) \approx \frac{1}{2}$$

So, looking at the graph,

$$\sin^2 2\theta \approx 1$$

So $\theta \approx \pi/4$, corresponding to a maximal mixing. The best fit (dashed line) to the data is for

$$\Delta m^2 \approx 3 \times 10^{-3} (eV/c^2)^2$$

<u>Neutrino Oscillations in Matter and Solar Neutrinos</u>

Now, remember that we wrote

$$E_i \approx \frac{m_i^2}{2p} + p$$

So, since the time evolution of mass eigenstates is given by

$$i\frac{d\psi}{dt} = E\psi$$

We can write

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}m_1^2/2p & 0\\ 0 & m_2^2/2p\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} + \begin{pmatrix}p & 0\\ 0 & p\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

We omit the last term since it is a constant factor that affects both states similarly. Then, since

$$\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

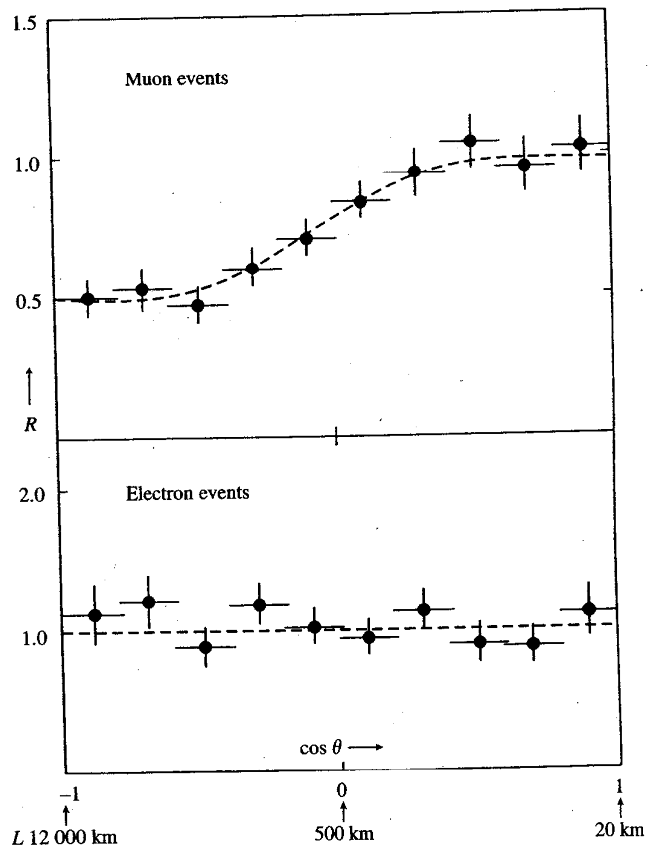


Fig. 6.17 Observed zenith angle distribution of electron and muon events with lepton momenta above 1.3 GeV/c in the Super-kamiokande detector, plotted as a ratio to the rate expected for no oscillations (Fukuda *et al* (1998)). The dashed curve in the upper plot shows the predicted variation for maximum $\nu_\mu \to \nu_\tau$ mixing with $\Delta m^2 = 3\times 10^{-3}$ eV^2. The lower plot indicates no oscillations of electron-neutrinos. The horizontal scale indicates the neutrino path lengths through the Earth and the atmosphere, at different zenith angles.

Figure 6.18 shows the calculated fluxes of neutrinos at the Earth as a functi[on] of energy. Although the fluxes at the higher energies, notably from ^{8}B dec[ay,] are very small compared with those from the pp reaction, they make substan[tial] contributions to the total event rate since the cross sections for the detect[ors] employed vary approximately as E_ν^3. Table 6.1 shows the results from sev[eral] experiments, giving the ratio of the observed event rate to that calculated [by] Bahcall *et al.* (2001) in the absence of oscillations. The first two entries are radiochemical experiments, detecting the accumulated activity of the pro[duct] nuclei after fixed time periods. They offered formidable experimental c[hal]lenges, requiring the detection of less than one atom of the product element [per] day, in a mass of 50 tons (in the case of gallium) or 600 tons (in the cas[e of] chlorine). The gallium experiments SAGE and GNO have a threshold en[ergy] of 0.2 MeV and are therefore sensitive to pp neutrinos, which from Fig. [...] extend up to an energy of 0.4 MeV.

The remaining experiments have higher thresholds and are not sensitive t[o] pp neutrinos. The SNO and SUPER-K experiments employ 1 kiloton of h[eavy] water and 30 kilotons of light water, respectively, and both measure e[vents] in real time, detecting the Cerenkov light emitted by the relativistic elec[trons] traversing the water using large photomultiplier arrays (see Fig. 3.13)[.] electrons originate from elastic scattering reactions (rows 4 and 5 Tabl[e]

We can write

$$i\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}=\begin{pmatrix}m_1^2/2p & 0\\ 0 & m_2^2/2p\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

Therefore

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}m_1^2/2p\cos\theta & -m_1^2/2p\sin\theta\\ m_2^2/2p\sin\theta & m_2^2/2p\cos\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

$$=\begin{pmatrix}m_1^2/2p\cos^2\theta+m_2^2/2p\sin^2\theta & \cos\theta\sin\theta(m_2^2/2p-m_1^2/2p)\\ \cos\theta\sin\theta\,(m_2^2/2p-m_1^2/2p) & m_1^2/2p\sin^2\theta+m_2^2/2p\cos^2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

$$=\begin{pmatrix}m_1^2/2p+\sin^2\theta\;\Delta m^2/2p & \frac{1}{2}\sin 2\theta\,(\Delta m^2/2p)\\ \frac{1}{2}\sin 2\theta\,(\Delta m^2/2p) & m_1^2/2p+\cos^2\theta\;\Delta m^2/2p\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

And since

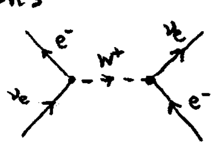
$$\sin^2\theta=\tfrac{1}{2}(1-\cos 2\theta),\quad \cos^2\theta=\tfrac{1}{2}(1+\cos 2\theta),\quad p\sim E$$

We get

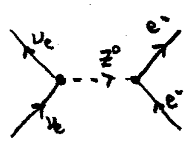
$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}=\left(\left[\frac{(m_1^2+m_2^2)}{4E}\right]\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\frac{\Delta m^2}{4E}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}\right)\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

$$=M_v\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

where the V stands for vacuum. But now, consider the case of MeV neutrinos traveling through matter. ν_e's can undergo charged and neutral current interactions



charged



neutral

but muon and tau neutrinos cannot participate in the charged interaction in the absense of muons and taus. Because of this, an extra potential V_e affects the forward scattering amplitude of electron neutrinos, changing their effective mass. We write

$$V_e=G_F N_e\sqrt{2}$$

where G_F is the Fermi constant and N_e is the electron density. Then

$$m_i^2=(E_i+V_e)^2-p^2\approx m^2+2E_iV_e$$

So the change in mass is

$$\Delta m_m^2=2\sqrt{2}\,G_F N_e E=2EV_e$$

Therefore

$$\frac{1}{2}(m_1^2+m_2^2)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\longrightarrow\frac{1}{2}(m_1^2+m_2^2)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+2\sqrt{2}\,G_F N_e E\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

$$=\left[\frac{1}{2}(m_1^2+m_2^2)+\sqrt{2}\,G_F N_e E\right]\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\sqrt{2}\,G_F N_e E\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

There, the matrix defining travel through matter is

$$M_M = \left[\frac{m_1^2+m_2^2}{4E} + \frac{\sqrt{2}\,G_F N_e}{2}\right]\begin{pmatrix}1&0\\0&1\end{pmatrix} + \frac{\Delta m^2}{4E}\begin{pmatrix}-\cos 2\theta + A & \sin 2\theta\\ \sin 2\theta & \cos 2\theta - A\end{pmatrix}$$

where

$$A = \frac{2\sqrt{2}\,G_F N_e E}{\Delta m^2}$$

Notice that the first term affects ν_e and ν_x similarly, so we can omit it. Now, following what we did for vacuum oscillations, we know that it must be true that we can write

$$\frac{\Delta m_m^2}{4E}\begin{pmatrix}-\cos 2\theta_m & \sin 2\theta_m\\ \sin 2\theta_m & \cos 2\theta_m\end{pmatrix} = \frac{\Delta m^2}{4E}\begin{pmatrix}-\cos 2\theta + A & \sin 2\theta\\ \sin 2\theta & \cos 2\theta - A\end{pmatrix}$$

where θ_m is a matter mixing angle. Then we can see that

$$\cos 2\theta_m = \frac{1}{A}(\cos 2\theta - A)$$

and

$$\sin 2\theta_m = \frac{1}{A}\sin 2\theta$$

So

$$\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta - A} = \frac{\tan 2\theta}{(1 - A\sec 2\theta)}$$

Now, note that we can get some sort of resonance if $A = \cos 2\theta$, which will correspond to $\theta_m = \pi/4$, maximal mixing in matter. Note that the condition is

$$\frac{2\sqrt{2}\,G_F N_e E}{\Delta m^2} = \cos 2\theta$$

or

$$N_e = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\,G_F E}$$

So we have a sort of resonance electron density. If the electron density is ever this value, we will observe maximal mixing of electron neutrinos to other neutrino states. So, we only need to ensure that the maximum density is greater than the resonance density; or, we can put this in terms of a minimum required energy to see resonance

$$E_{min} = \Delta m^2 \frac{\cos 2\theta}{2\sqrt{2}\,G_F N_{e(max)}}$$

So, we would expect to see a higher suppression of electron neutrinos with higher energy neutrinos.

The situation we have just gone through describes precisely what happens to neutrinos as they travel through the sun. The nuclear fusion process that powers the sun produces neutrinos with a spectrum of energies. Large neutrino observatories have been operating since the 1960s to observe these neutrinos, but they recorded a deficit of neutrinos received compared to the predicted flux from solar models. This "Solar Neutrino Problem" was unsolved for about 40 years, until results from the Sudbury Neutrino Observatory (SNO) offered compelling evidence for neutrino flavor oscillations. SNO utilized two interactions of neutrinos with deuteron nuclei

$$\nu_e + d \rightarrow e^- + p + p \quad \text{(charged current)}$$

$$\nu_x + d \rightarrow \nu_x + p + n \quad \text{(neutral current)}$$

The charged current reaction only occurs for electron neutrinos while the neutral current interaction occurs for all three flavors. SNO observed that

Interaction	observed/expected rate
CC	0.35 ± 0.03
NC	1.01 ± 0.12

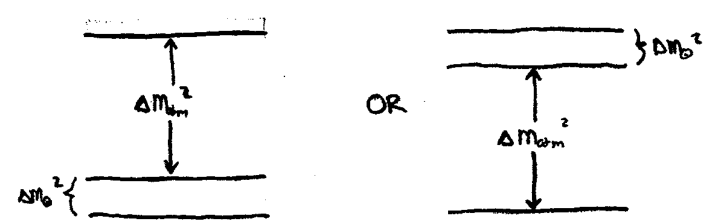
So, SNO's results are compatible with the idea that neutrinos have mass and oscillate. Data from solar neutrinos gives a best fit for oscillation of $\nu_e \rightarrow \nu_x$ with

$$\Delta m_\odot^2 \sim 7 \times 10^{-5} \, (eV/c^2)^2$$

Recall that atmospheric neutrinos gave a mass difference of

$$\Delta m_{atm}^2 \sim 3 \times 10^{-3} \, (eV/c^2)^2$$

So we get a picture that most likely looks like this:



We cannot be sure of the order: the first represents a normal heirarchy; the second is an inverted heirarchy.

The LSND Experiment and Sterile Neutrinos

Finally, we should mention the LSND experiment at Los Alamos. The experiment studied muon anti-neutrinos from the decay of anti-muons at rest

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

The detector, located about 30 m away from the source, measured the oscillation of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, and measured an excess of $\bar{\nu}_e$ above background due, supposedly, to the oscillation of $\bar{\nu}_\mu$. But the best fit of the data gives

$$\Delta m^2 \sim 0.5 \, (eV/c^2)^2$$

This presents a problem: it has been shown from Z^0 decay that there are only three flavors of neutrinos that couple to the weak force. But we have three different Δm^2 values of three different orders of magnitude. So, if the LSND experimental result is correct, there must be at least four different neutrino mass eigenstate, which would mean there is at least one neutrino flavor that does not couple to the weak force. Neutrinos can oscillate into these "sterile" states, but cannot be detected by conventional means.

The experiment MiniBoone at Fermilab has duplicated LSND, and results should be released any day now. The possibility of sterile neutrinos is an exciting break from the Standard Model.

Sources

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