

6.3: Feynman rules for toy theory.

- Each line and vertex in Feynman Diagram represent mathematical terms
- The probability of a certain interaction is calculated by drawing the corresponding Feynman Diagram
- The probability [Amplitude] contains all dynamical information, enables us to calculate the 'Cross section, Life time'.

Feynman showed how to calculate [\mathcal{M} : amplitude] using "Feynman Rules":

1. Notation.

- Label incoming and outgoing four momenta
 p_1, p_2, \dots
- Label the ~~intermediate~~ internal momenta q_1, q_2, \dots

Lines present "Real Particles" (i.e. that can be observed, they are the initial and final particles)

internal lines present "virtual particles" (i.e. can't be observed, do not obey $m^2 c^4 = E^2 - (p^2 c^2)$)
They can be considered a kind of "Quantum tunneling"

2. Coupling constant:

for each vertex, there is a factor $[-ig]$

The Hamiltonian of a system can be separated into Kinetic part and Interaction part

g : determines the strength of the interaction part (e.g. charge is a coupling constant).

3. Propagator:

for each internal line.

- it gives the probability for a particle to travel through space with a certain Energy & Momentum.

$$\left[\frac{i}{q^2 - m^2 c^2} \right] \quad q^2 = q^\mu q_\mu$$

- $q^2 \neq m^2 c^2$, because virtual particles do not lie on their mass shell, they are allowed to be off shell (i.e. not to satisfy classical equation of motion)

4. Conservation of Energy and Momentum.

for each vertex there is a "Delta function".

$$\left[(2\pi)^4 \delta^4(k_1 + k_2 + k_3) \right]$$

$k_1, k_2, \text{ and } k_3$ are the four momenta coming into the vertex.

(if arrows lead outward, k is minus).

5. Integration over Internal Momenta.

for each internal line, we will integrate all over internal momenta, with a factor.

$$\left[\frac{1}{(2\pi)^4} d^4 q_j \right]$$

6. Cancel the Delta Function.

The result of integration will include a delta function

$$(2\pi)^4 \delta^4(p_1 + p_2 + \dots - p_n).$$

This factor will be erased, that we will only have

$$\left[-i\mathcal{M} \right] \text{ the amplitude}$$
