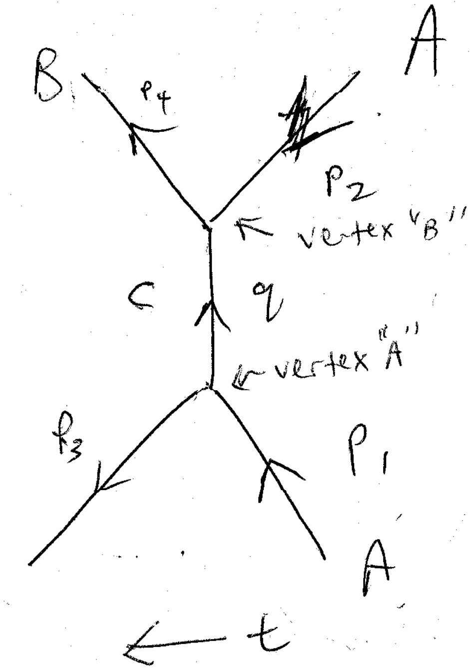
$\boxed{6.5}$

by WOOBK



$A + A \rightarrow B + B$

31 Oct

Two vertices

So $(\cancel{2})(-ig)(-ig)$

one internal line

$$1 \times \frac{i}{q_j^2 - m_j^2 c^2} \quad \text{rule 3}$$

1 vertices (vertex "A") $\Rightarrow (2\pi)^4 \delta^4(p_1 - p_3 - q)$ (rule 2)

1 vertices (vertex "B") $\rightarrow (2\pi)^4 \delta^4(q + p_2 - p_4)$ (rule 2)

Rule 5

$$-iM = \int (-ig)_{A}(-ig) \cancel{(2\pi)^4} \delta^4(p_1 - p_3 - q)(2\pi)^4 \delta^4(q + p_2 - p_4) \frac{i}{q^2 - m_c^2 c^2} \frac{d^4 q}{\cancel{(2\pi)^4}}$$

$\boxed{i^2 = -1}$

$$-iM = +\int +i^2 g^2 \delta^4(p_1 - p_3 - q)(2\pi)^4 \delta^4(q + p_2 - p_4) \frac{i}{q^2 - m_c^2 c^2} \frac{d^4 q}{1}$$

$$M = \int g^2 \delta^4(p_1 - p_3 - q)(2\pi)^4 \frac{\delta^4(q + p_2 - p_4)}{q^2 - m_c^2 c^2} d^4 q$$

$q = p_4 - p_2$

$$\int_{-\infty}^{\infty} f(x)\delta(x-p)\,dx = f(p)$$

$$M = g^2 \frac{\cancel{\delta^4(p_1 - p_3 - p_4 + p_2)(2\pi)^4}}{(p_4 - p_2)^2 - m_c^2 c^2}$$

(amplitude)

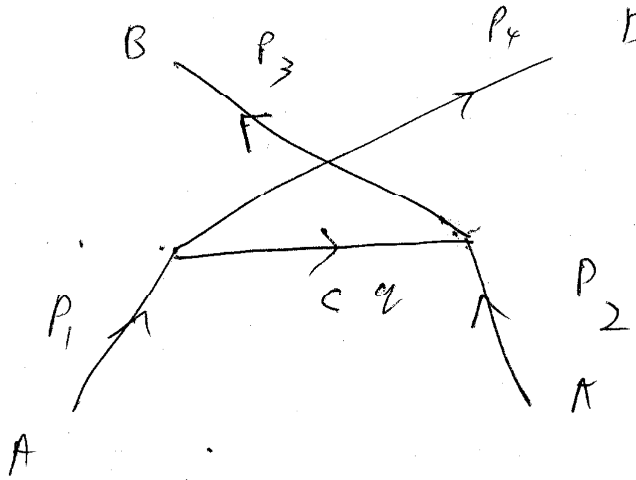
due to rule 6.

1/6

then rule 6 gives us :

$$\mathcal{M} = \frac{g^2}{(p_4 - p_2)^2 - m_c^2 c^2} \quad (6.49)$$

interchange
$p_4 \leftrightarrow p_3$



gives us

$$\mathcal{M} = \underbrace{\frac{g^2}{(p_4 - p_2)^2 - m_c^2 c^2}} + \underbrace{\frac{g^2}{(p_3 - p_2)^2 - m_c^2 c^2}} \quad (6.50)$$

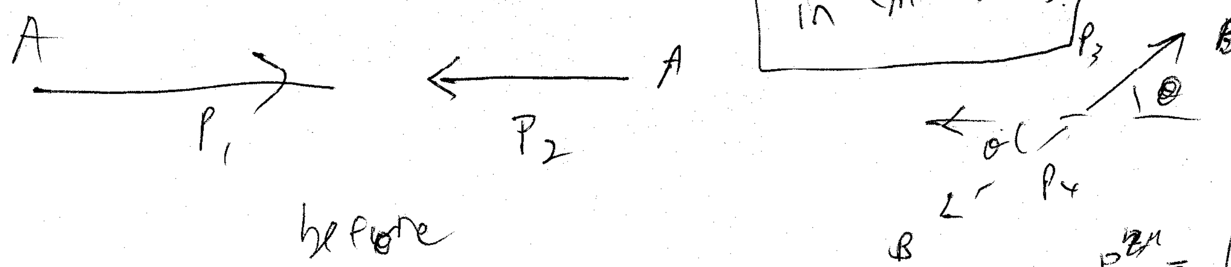
$$(p_4 - p_2)^2 - m_c^2 c^2$$

$$= p_4^2 + p_2^2 - 2 p_2^{\mu} \cdot p_{4\mu} - \underbrace{m_c^2 c^2}_{0} \quad \text{if } m_c = 0$$

$$= p_4^2 + p_2^2 - \underbrace{2 p_2^{\mu} \cdot p_{4\mu}}_{\text{need to evaluate !!!}}$$

2/6

in cm frame!!



$$p_2^{\mu} = \begin{pmatrix} \frac{E_2}{c} \\ \vec{p}_2 \end{pmatrix} = p_2^{\mu}$$

$$p_{4\mu} = \begin{pmatrix} \frac{E_4}{c} \\ -\vec{p}_4 \end{pmatrix}$$

$$p_2^{\mu} p_{4\mu} = \frac{E_2 E_4}{c^2} - \vec{p}_2 \cdot \vec{p}_4$$

$$= \frac{E_2 E_4}{c^2} - |\vec{p}_2||\vec{p}_4| \cos\theta$$

if $m_A = m_B = m$ $\Rightarrow$

use

$$\boxed{p^A p_A = m_A^2 c^2}$$

$$\boxed{|\vec{p}| = |\vec{p}_2| = |\vec{p}_4|}$$

use

$$\boxed{E^2 = m^2 c^4 + \vec{p}^2 c^2}$$

$$\therefore \cancel{2 p_2 \cdot p_4} = \frac{E^2}{c^2} - \vec{p}^2 \cos\theta$$

$$p_2^{\mu} p_{4\mu} = \frac{E^2}{c^2} - \vec{p}^2 \cos\theta$$

$$-2 p_2^{\mu} p_{4\mu} = -2\frac{E^2}{c^2} + 2|\vec{p}|^2 \cos\theta$$

$$\boxed{|\vec{p}| = |\vec{p}_2| = |\vec{p}_4|}$$

$$p_4^2 + p_2^2 - 2(p_2^{\mu} p_{4\mu})$$

$$= 2m^2c^2 - \frac{2E^2}{c^2} + 2|\vec{p}|^2 \cos\theta$$

$$= -2\left[\left(\frac{E^2}{c^2} - m^2c^2\right) - |\vec{p}|^2 \cos\theta\right]$$

$$= -2\left[|\vec{p}|^2 - |\vec{p}|^2 \cos\theta\right]$$

$$= -2|\vec{p}|^2 (1 - \cos\theta] \quad \# \quad (6.51)$$

$$(P_3 - P_2)^2 - M_c^2 c^2$$

$M_c = 0$

$$= P_3^2 + P_2^2 - 2 P_3^{\mu} \cdot P_{2\mu} - \cancel{M_c^2 c^2}^{\,0}$$

$$= P_3^2 + P_2^2 - 2 P_3^{\mu} \cdot P_2$$

$$P_3^{\mu} \cdot P_{2\mu} = \frac{E_3 E_2}{c^2} - \underbrace{\vec{P}_3 \cdot \vec{P}_2}_{= -\cos\theta}$$

$$= \frac{E^2}{c^2} - |\vec{P}_3||\vec{P}_2| \cos(\pi - \theta)$$

$$= \frac{E^2}{c^2} + |\vec{p}|^2 \cos\theta$$

$$\therefore P_3^2 + P_2^2 - 2 P_3^{\mu} \cdot P_{2\mu} = 2 m^2 c^2 - \frac{2E^2}{c^2} - 2|\vec{p}|^2 \cos\theta$$

$$= -2\left[\left(\frac{E^2}{c^2} - m^2 c^2\right) + |\vec{p}|^2 \cos\theta\right]$$

$$= -2\left[\vec{p}^{\,2} + |\vec{p}|^2 \cos\theta\right]$$

$$= -2|\vec{p}|^2 (1 + \cos\theta) \quad \cancel{\#} \; (6.52)$$

so

$$\mathcal{M} = \frac{g^2}{-2|\vec{p}|^2((1-\cos\theta)} + \frac{g^2}{-2|\vec{p}|^2(1+\cos\theta)}$$

$$= \frac{g^2}{-2|\vec{p}|^2}\left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}\right)$$

$$= \frac{g^2}{-2|\vec{p}|^2}\left(\frac{1+\cos\theta + 1-\cos\theta}{1-\cos^2\theta}\right)$$

$$\mathcal{M} = \frac{g^2}{-2|\vec{p}|^2}\frac{(2)}{\sin^2\theta} = \frac{-g^2}{|\vec{p}|^2\sin^2\theta} \quad \#\# \qquad (6.5)$$

from $$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2}\frac{|\vec{p}_f|}{|\vec{p}_i|} \qquad (6.42)$$

$$\therefore \frac{d\sigma}{d\Omega} =$$

$S = \frac{1}{2}$ *

5/6

$$S = \frac{1}{j!} = \frac{1}{2!}$$

no. of identical particles?

$$S = \frac{1}{2}$$

$j = 2$

so $|\mathcal{M}|^2 = \dfrac{g^4}{|\vec{p}|^4 \sin^4\theta}$

From (6.42):

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \left(\frac{1}{2}\right) \frac{g^4}{|\vec{p}|^4 \sin^4\theta} \frac{}{(E_1+E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

$= 1$

& $\boxed{E_1 = E_2 = E}$

$$= \frac{1}{2}\frac{1}{32}\left(\frac{\hbar c}{\pi}\right)^2\left(\frac{1}{2}\right)\frac{g^4}{|\vec{p}|^4 \sin^4\theta \; 4E^2}$$

$$= \frac{1}{2}\left(\frac{\hbar c g^2}{16\pi E^2 |\vec{p}|^2 \sin^2\theta}\right)^2$$

(6.54)

$64 \times 4 = 16^2$

$16 \times 4 \times 4 = 16^2$

6/6