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PHYS 4213
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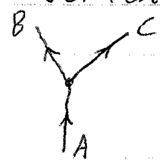
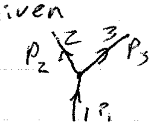
Undergrad Presentation of Section 6.4

Section 6.4 covers the simplest solutions for finding decay rate Γ and lifetime τ for a single particle decay. The proof is based in Griffiths' "Toy Theory" introduced in Sec. 6.3. This "simplest solution" is a first order approximation of Fermi's Golden rules. By "first order" I mean we will consider a decay which has only one possible decay (strong, weak, or EM) and we will only consider the direct decay which has no internal lines and a single vertex:



First of we will apply the Golden rules for a simple decay as shown in example 6.6 of section 6.1. What follows is a review of that example:

Given



$$d\Gamma = |M|^2 \frac{S}{2\hbar m_1}\left[\left(\frac{c\,d^3p_2}{(2\pi)^3 2E_2}\right)\left(\frac{c\,d^3p_3}{(2\pi)^3 2E_3}\right)\right](2\pi)^4 \delta^4(p_1-p_2-p_3)$$

where p_i are four-vector momenta
where $|M|$ is the amplitude of the reaction, which will be found later.

In this problem we will take the rest frame of particle, so that $\vec{p}_1$ can be eliminated and $|\vec{p}| \equiv |\vec{p}_2| = |\vec{p}_3|$ since momentum must be conserved. We will split the delta functions of the four vectors: $\delta^4(p_1-p_2-p_3) = \delta(E_1/c - E_2/c - E_3/c)\,\delta^3(\overset{0}{\cancel{\vec{p}_1}} - \vec{p}_2 - \vec{p}_3)$ and replace $d^3p = |\vec{p}|^2 d\vec{p}\, d\Omega$.

We will then integrate over all space $d\Omega$ and momenta dp.
The delta function will select the particular momentum such that $E_1 = m_1 c^2$ since we are in particle 1's rest frame:

$$|p| = \frac{c}{2m_1}\sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2 m_2^2 - 2m_1^2 m_3^2 - 2m_2^2 m_3^2}$$

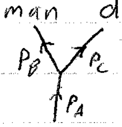
The integral is the solution:

$$\int d\Gamma = \Gamma = \frac{1}{\tau} = \boxed{\frac{S\,|p|\,|M|^2}{8\pi\hbar\, m_1^2\, c}} \qquad \text{Eqn. ①}$$

Which is the result for a general decay into particles with any mass.

Next we will apply rules for determining $|M|$ for our particular case $A \rightarrow B + C$

1) draw Feynman diagram



2) assign a factor of $-ig$ for each vertex
   - we have one vertex.

3) propagators
   - we have none

4) assign $(2\pi)^4 \delta^4(k_{in} + \ldots - k_{out} - \ldots)$ for all ~~external lines and internal lines~~ @each vertex
   - $(2\pi)^4 \delta^4(p_A - p_B - p_C)$

5) internal lines
   - we have none

6) truncate delta functions for external lines.
   - $(2\pi)^4 \delta^4(p_A - p_B - p_C)$ truncated

The factors which remain will $= -iM$

$$-iM = -ig\,\cancel{(2\pi)^4 \delta^4(p_A - p_B - p_C)} = -ig$$

$$\boxed{M = g} \qquad \text{Eqn. ②}$$

Now we will plug this solution into Eqn. 1 in order to evaluate our simple decay:

$$① \longrightarrow \Gamma = \frac{1}{\tau} = \boxed{\frac{g^2 |p|}{8\pi \hbar m_A^2 c}} \qquad \text{Eqn. } ⑤$$

by replacing $m_A = m_1$ and $S = 1$ since B and C are distinguishable particles.

Now we will work Problem 6.6 (Griffiths):

Consider the decay $\pi^0 \longrightarrow \gamma + \gamma$

Assume π^0 is an elementary particle so that the toy theory is applicable. Approximate the lifetime of the π^0 by using Eqn. 6.23 and using dimensional analysis to make a guess for the formulation of $|M|$.

Eqn. 6.23 ~~___~~ is the case of a decay into two massless particles ($\gamma + \gamma$ for example):

$$\tau = \frac{16\pi \hbar m_1}{S |M|^2} \qquad ③$$

Eqn. 6.44 will tell us the dimensions $|M|$

$$[M] = (mc)^{4-n} \text{ where } n \text{ is \# ext. lines.}$$

In our case $n=3$ so $[M]$ = units of momentum.

Using the information given in the problem there is one straightforward manner to construct units of momentum: $(m_{\pi^0} c)$

So we will assume the form

$$|M| = \alpha_{EM} \, m_{\pi^0} c \quad \text{since the decay is EM.}$$

Plugging this result into ③ we obtain

$$\tau = \frac{32\pi\hbar}{\alpha_{EM}^2 m_{\pi^0} c^2} = 9.2 \times 10^{-18} s$$

which is one order of magnitude from the experimental value $8.7 \times 10^{-17} s$

Note that the previous example is the only case which will work via this method b/c all decays are more complicated than the toy theory, most decays have secondary decays which aren't accounted for in this simplification, and a decay to massive particles would leave the reader guessing at the form of $|M|$.

The next simple decay problem are $n \rightarrow p + e^- + \bar{\nu}_e$ and $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ which are discussed in Sections 10.2 and 10.3.

Note that no decays in nature are simple enough to be approximated by Eqn. 5