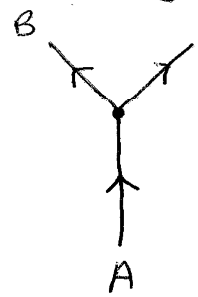
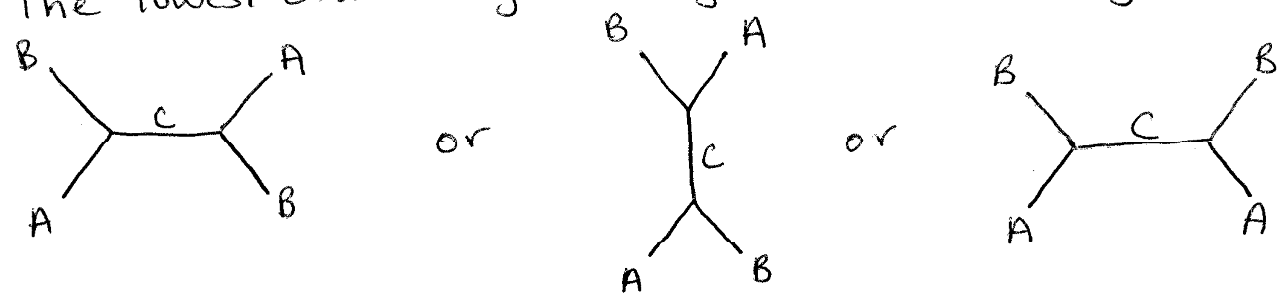
6.3 The Feynman Rules for a Toy Theory (ABC Theory)

- Start with 3 particles; A, B, and C
  - with masses m_A, m_B, and m_C
  - each with spin 0 (particles with spin are dealt with in Chapter 7)
  - each it's own antiparticle (so the arrow denotes whether its incoming or outgoing in the process only)
  - let A be the heaviest of the 3 particles
- The lowest order Feynman Diagram for decay is



- The lowest order Feynman Diagrams for Scattering are

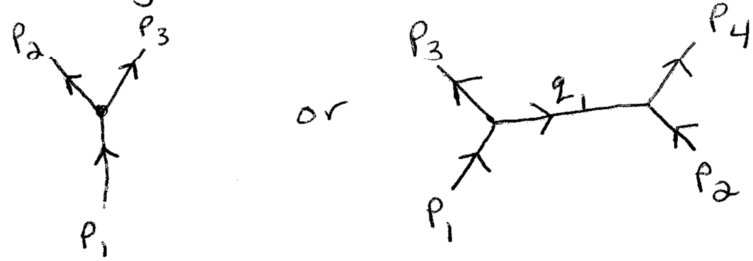


- We'll deal with higher order diagrams in Sec. 6.6
- What we are trying to do is calculate $\mathcal{M}$ which is the amplitude of the process.
- We need $\mathcal{M}$ to calculate the
  - decay rate
  - Cross section for scattering

Kat Baker
P1/3

To Calculate $\mathcal{M}$ we have 6 rules

1) <u>Notation</u>: Draw the Feynman diagram, label each external line with its four-vector momenta $p_1, p_2, p_3 \ldots$ and each internal line with its four-vector momenta $q_1, q_2 \ldots$ Put arrows on each line to keep track of positive/negative. (incoming - positive, outgoing - negative)



2) <u>Coupling Constant</u> - For each vertex, write down a factor of $-ig$

g is the coupling constant. It specifies the strength of the interaction between A, B, and C. $\left(\alpha = \frac{g^2}{4\pi} \right)$

3) <u>Propagator</u> - For each internal line, write down a factor of $\frac{i}{(q_j^2 - m_j^2 c^2)}$

4) <u>Conservation of Energy and Momentum</u> - For each vertex, write down $(2\pi)^4 \delta^4 (k_1 + k_2 + k_3)$

k is the 4 vector momenta, either p_i or q_j

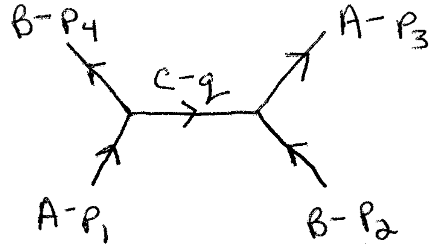
5) <u>Integration over Internal Momenta</u> - For each internal line, write down $\frac{1}{(2\pi)^4} d^4 q_j$ 5b) Then Integrate

6) <u>Cancel the Delta Function</u> - Erase $(2\pi)^4 \delta^4 (p_1 + p_2 + \ldots - p_n)$

What you are left with $= -i\mathcal{M}$

Example $A+B \to A+B$

1)



2) We have 2 vertices $\Rightarrow \quad (-ig)(-ig) = -g^2$

3) 1 internal line $\Rightarrow \quad \dfrac{i}{q^2 - m_c^2 c^2}$

4) 2 vertices $\Rightarrow \quad (2\pi)^4 \delta^4(p_1 - p_4 - q)(2\pi)^4 \delta^4(q + p_2 - p_3)$

5) 1 internal line $\Rightarrow \quad \dfrac{1}{(2\pi)^4} d^4 q$

5b) Multiply and Integrate

$$\int (-g^2)\left(\frac{i}{q^2 - m_c^2 c^2}\right)(2\pi)^4 \delta^4(p_1 - p_4 - q)(2\pi)^4 \delta^4(q + p_2 - p_3) \frac{1}{(2\pi)^4} d^4 q$$

$$= \int \frac{-g^2 i}{q^2 - m_c^2 c^2} (2\pi)^4 \delta^4(p_1 - p_4 - q)\delta^4(q + p_2 - p_3)\, d^4 q$$

Integrating over the Delta Function gives us $q = p_1 - p_4$

$$= \frac{-g^2 i}{(p_1 - p_4)^2 - m_c^2 c^2} (2\pi)^4 \delta^4(p_1 - p_4 + p_2 - p_3)$$

6) $$= \frac{-g^2 i}{(p_1 - p_4)^2 - m_c^2 c^2} = -i\mathcal{M}$$

$$\mathcal{M} = \frac{g^2}{(p_1 - p_4)^2 - m_c^2 c^2} = \frac{g^2}{(p_A - p_B)^2 - m_c^2 c^2}$$