Stellar Nucleosynthesis

I The nature of stars & the sun

1. ~1 million miles in diameter
2. ~91% H ~9% He ~1% other "metals"
   (% of total number of atoms)
3. The core
   Size $\sim\frac{1}{4} R_\odot$
   density $160 \times 10^3\ kg/m^3$
   Temp. 14 million K

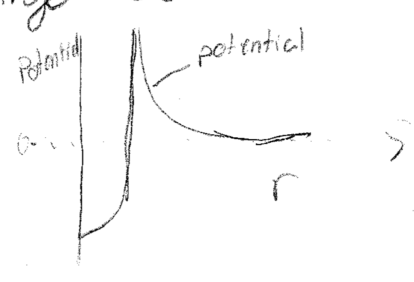
II Energy Released by a Nuclear Reaction
found from mass defect
= mass of particles going into reaction - mass of particles created

Proton-Proton chain - more later
$4p \rightarrow 1\alpha$
$4.0312 - 4.0026 = .0286\,u = 26.6\ MeV$

III Reactions - Rates + Probabilities
1 Most reactions in the sun require tunneling
p + nucleus means that there is a coulomb barrier with potential energy $\frac{e^2}{4\pi\epsilon_0 r}$. For fusion to take place, the $\frac{1}{2}mv^2$ kinetic energy must exceed the potential at small r

$r \leq \frac{2e^2}{4\pi\epsilon_0 m_p v^2}$ $\quad \frac{1}{2}mv^2 = \frac{3}{2}kT \quad v = \left(\frac{3kT}{m}\right)^{1/2}$

v must be very large so T must be huge



2. Reaction Rates

Particle a collides with nucleus X to produce nucleus Y and particle b

$a + X \rightarrow Y + b$

$dn_a(v_a)$ number of partices a in $1cm^3$ in volume element d^3v_a in the velocity space

$dn_x(v_x)$

$v = v_a - v_x$ relative velocity for 2 particles

$\sigma(v)$ reaction crossection (Probability of interaction between the incident particle and the target nucleus)

r_{ax} · number of nuclear reactions in $1cm^3$ per second

$$r_{ax} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sigma(v)\, v\, dn_a(v)\, dn_x(v) \quad (1)$$

The speed of the particles follow the maxwell distribution

$$dn_i(v_i) = n_i \left(\frac{m_i}{2\pi kT}\right)^{3/2} e^{\frac{-m_i v_i^2}{2kT}} d^3v_i \quad (2) \qquad \text{put (2) into (1)}$$

$$r_{ax} = n_a n_x \left(\frac{m_a}{2\pi kT}\right)^{3/2}\left(\frac{m_x}{2\pi kT}\right)^{3/2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\left(\frac{m_a v_a^2}{2kT} + \frac{m_x v_x^2}{2kT}\right)} \sigma(v)\, v\, d^3v_a\, d^3v_x \quad (3)$$

if U is the velocity of the COM of the two particles

$$(m_a + m_x)U = m_a v_a + m_x v_x \qquad v = v_a - v_x$$

$$\Rightarrow \quad v_a = U + \frac{m_x}{m_a + m_x} v \qquad v_x = U - \frac{m_a}{m_a + m_x} v$$

$$\mu = \frac{m_x m_a}{m_a + m_x} \qquad \frac{1}{2} m_a v_a^2 + \frac{1}{2} m_x v_x^2 = \frac{1}{2}(m_a + m_x)U^2 + \frac{1}{2}\mu v^2 \quad (4)$$

Put (4) into (3) and $d^3v_a \cdot d^3v_b = d^3U \cdot d^3v = 4\pi U^2 dU \cdot 4\pi v^2 dv$

$$r_{ax} = n_a n_x \left(\frac{m_a}{2\pi kT}\right)^{3/2}\left(\frac{m_x}{2\pi kT}\right)^{3/2} \int_0^{\infty}\int_0^{\infty} e^{-\left(\frac{(m_a + m_x)U^2}{2kT} + \frac{\mu v^2}{2kT}\right)} \sigma(v)\, v\, 4\pi U^2 dU \cdot 4\pi v^2 dv \quad (5)$$

$$\int_0^{\infty} e^{-\frac{(m_a + m_x)U^2}{2kT}} 4\pi U^2 dU = \left(\frac{2\pi kT}{m_a + m_x}\right)^{3/2}$$

$$r_{ax} = 4\pi n_a n_x \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int_0^{\infty} e^{-\frac{\mu v^2}{2kT}} \sigma(v)\, v^3 dv \quad (6)$$

$E=\frac{1}{2}\mu v^2$ (6) becomes

$$r_{ax}=n_a n_x\int_0^\infty f(E)\sigma v\,dE = n_a n_x\langle\sigma v\rangle \quad (7)$$

$$\langle\sigma v\rangle=\int_0^\infty f(E)\sigma v\,dE \qquad f(E)=\frac{2}{\pi^{1/2}}\frac{1}{(kT)^{3/2}}e^{-\frac{E}{kT}}E^{1/2}$$

if a and X are the same then the probability of having a collision $\propto \frac{n_a(n_a-1)}{2}\approx\frac{n_a^2}{2}=\langle\sigma v\rangle$

so $r_{ax}=\frac{1}{1+\delta_{ax}}n_a n_x\langle\sigma v\rangle$

3. Reaction Probability $\langle\sigma v\rangle$

$$\sigma v=\frac{r_{ax}}{dn_a\,dn_x}$$

dn_i number of particles in $1\,cm^3$

r_{ax} is nuclear reaction rate

= product of two probabilities P_{coul}. P_{nu}

P_{coul} = probability of tunneling = (

P_{nu} = probability of the particle having a reaction in the nuclear force range

$$P_{coul}=\left(\frac{2}{m}\right)^{1/2}E^{1/2}e^{-B/E^{1/2}} \qquad B=\frac{4\pi^2 Z_1Z_2e^2}{h}\left(\frac{m}{2}\right)^{1/2}$$

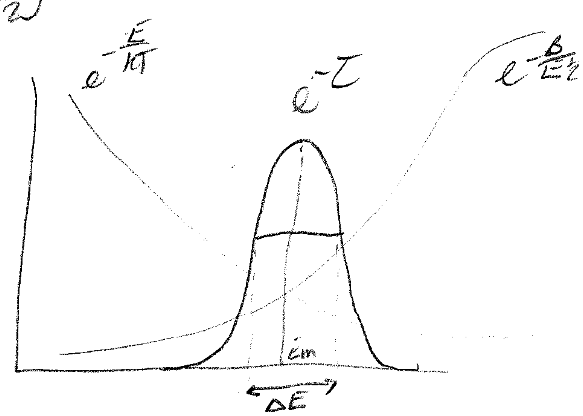
$\sigma v \propto P_{coul}$ for non-resonance

$$\langle\sigma v\rangle\propto\left(\frac{2}{\pi m}\right)^{1/2}\frac{2}{(kT)^{3/2}}\int_0^\infty e^{-\frac{E}{kT}-\frac{B}{E^{1/2}}}dE$$

$$\tau=\frac{3E_m}{kT} \qquad \Delta E=\frac{\sqrt{8}E_m}{\tau^{1/2}}$$

$$\int e^{-E/kT-\frac{B}{E^{1/2}}}dE\approx\frac{\sqrt{8}E_m}{\tau^{1/2}}e^{-\tau}$$

$$\langle\sigma v\rangle\propto\frac{1}{T^{3/2}}e^{-\tau}$$



The integrand is known as the Gamow Peak

$E=E_{max}$ at the peak

ΔE is the half width

H- burning - can start at 10 million K

2 main processes

average time for each step to occur

PPI

a) $\cancel{H}^{p} + \cancel{H}^{p} \rightarrow {}^2H + e^+ + \nu_e$ (releases 1.442 MeV) ~6 billion yrs

b) ${}^2H + \cancel{H}^{p} \rightarrow {}^3He + \gamma$ 5.49 MeV ~6s $4\,{}^1H \rightarrow {}^4He + 4\gamma$

c) ${}^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H$ 12.86 MeV 10^6 yrs +26.72 MeV

d) $e^+ + e^- \rightarrow \gamma + \gamma$ 1.02 MeV instant (.26 MeV/ν_e)

a) is weak $\sigma < 10^{-51} m^2$ ($10^{-21} fm^2$) at $T \sim 1.5 \times 10^7 K$

too small to be measured, calculated from theory

b)+c) involve Coulomb barrier penetration σ much much larger

b) EM c) strong

At these densities ($10^5 kg/m^3$) d has a mean life of 1s

${}^3He$ " " " 2×10^5 yrs

reaction A controls the production of He

this chain accounts for 91% of reactions in the sun

PPII

a)

b)

c) ${}^3_2He + \alpha \rightarrow {}^7_4Be + \gamma$ + 1.59 MeV EM

Berillium

d) ${}^7_4Be + e^- \rightarrow {}^7_3Li + \nu_e$ + .861 MeV weak $\}$ $p + {}^3_2He + e^- \rightarrow \alpha$

e) ${}^7_3Li + p \rightarrow \alpha + \alpha$ + 17.35

.8 MeV/ν_e lost

PPIII

a)

b)

c)

Boron

d) ${}^7_4Be + p \rightarrow {}^8_5B + \gamma$ + .135 MeV

e) ${}^8_5B \rightarrow {}^8_4Be^* + e^+ + \nu_e$ + 14.02 MeV - excited β^+ decay (93% of time)

f) ${}^8_4Be^* \rightarrow \alpha + \alpha$ + 3.03 MeV

g) $e^- + e^+ \rightarrow \gamma + \gamma$ + 1.02 MeV

7.2 MeV/ν_e

CNO important for high mass stars $> 8 M_\odot$

$p + {}^{12}_{6}C \rightarrow {}^{13}_{7}N + \gamma + 1.94$ MeV   Slowest

${}^{13}_{7}N \rightarrow {}^{13}_{6}C + e^{+} + \nu_e + 1.20$ MeV β^{+} decay

$p + {}^{13}_{6}C \rightarrow {}^{14}_{7}N + \gamma + 7.55$ MeV   concentration of N in heavy stars

$p + {}^{14}_{7}N \rightarrow {}^{15}_{8}O + \gamma + 7.29$ MeV   slow

${}^{15}_{8}O \rightarrow {}^{15}_{7}N + e^{+} + \nu_e + 1.74$ MeV β^{+} decay

$p + {}^{15}_{7}N \rightarrow {}^{12}_{6}C + \alpha + \underline{4.96}$ MeV

$(e^{+} + e^{-} \rightarrow \gamma + \gamma + 1.02 \text{ MeV}) \times 2$ 26.72 MeV $4p \rightarrow 1\alpha + 2\nu_e$ 1.71 MeV/α in ν_e

He burning 100 Million K 10^8 kg/m^3

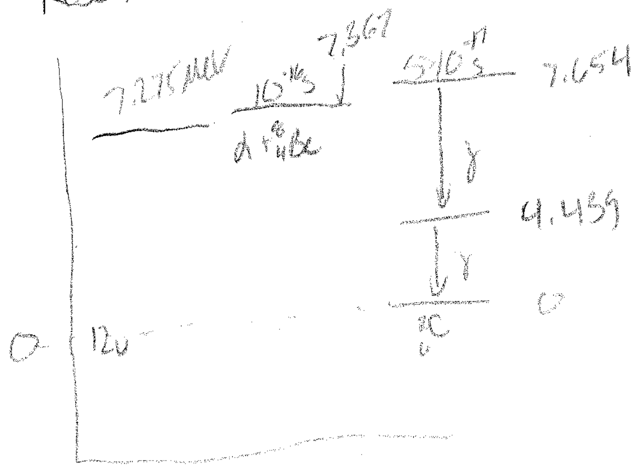
triple α process

a) $\alpha + \alpha \leftrightarrow {}^{8}_{4}Be - .09$ MeV

b) ${}^{8}_{4}Be + \alpha \rightarrow {}^{12}_{6}C + 2\gamma + 7.37$ MeV

${}^{8}_{4}Be$ has a mean life of 10^{-16} s so a) leads to an equilibrium concentration of ${}^{8}_{4}Be \rightarrow N_\alpha \sim 10^{-9}\ {}^{8}_{4}Be$

Resonance in reaction b) at COM Energy @ 287 keV $\rightarrow {}^{12}_{6}C$ excited $\rightarrow$ photoemission



c) ${}^{12}_{6}C + \alpha \rightarrow {}^{16}_{8}O + \gamma + 7.16$ MeV

This is the last thing the Sun can produce. it will end its life as a C:O White Dwarf

For more massive stars ...

$T \geq 7 \times 10^8$

${}^{12}_{6}C + {}^{12}_{6}C \rightarrow {}^{20}_{10}Ne + \alpha$, ${}^{23}_{11}Na + p$, ${}^{23}_{12}Mg + n$, ${}^{24}_{12}Mg + \gamma$, ${}^{16}_{8}O + 2\alpha$ A = 20-24

Ne $\rightarrow$ $T \geq 1.5 \times 10^9$ K

O burning $\rightarrow$ A = 24-32 Mg, Si, P, S. $T \geq 2 \times 10^9$ K

Si burning $\rightarrow$ Iron-like

3×10^9 K