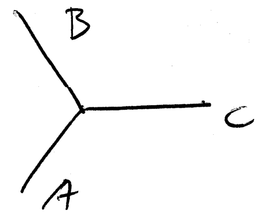
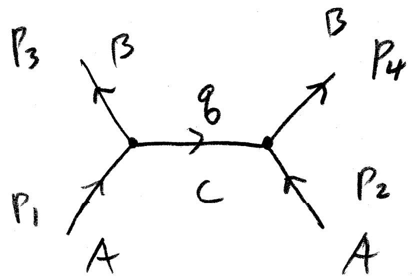
presentation – ~~[illegible]~~ Bahman

ch. 6 (con't)

fundamental diagram



$A + A \to B + B$



$$-iM = (-ig)(-ig)\frac{i}{q^2 - m_c^2}(2\pi)^4\delta^4(p_1 - p_3 - q)$$

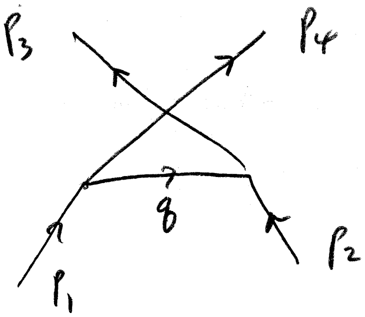
$$\cdot\ (2\pi)^4\delta^4(p_2 - p_4 + q)\frac{d^4q}{(2\pi)^4}$$

$p_2 - p_4 + q = 0$

$\Rightarrow\ q = p_4 - p_2$

$$-iM = (-ig)^2\frac{i}{(p_4-p_2)^2 - m_c^2}(2\pi)^4\delta^4(p_1 - p_3 - p_4 + p_2)$$

$$M = \frac{g^2}{(p_4-p_2)^2 - m_c^2}$$



Is another diagram that contributes.

Get another term in M with $p_3 \leftrightarrow p_4$:

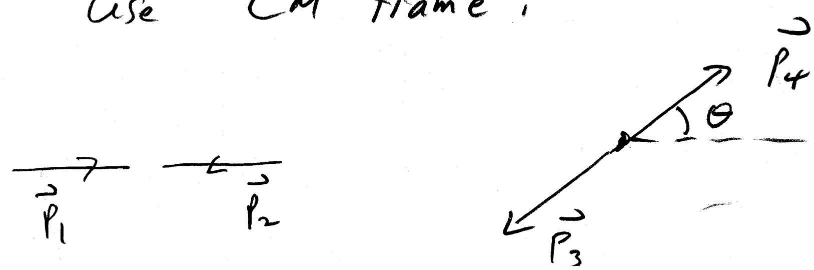
$$M = \frac{g^2}{(p_4 - p_2)^2 - m_c^2} + \frac{g^2}{(p_3 - p_2)^2 - m_c^2}$$

We derived :

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S\,|M|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

Assume : $m_A = m_B \equiv m$ and $m_c = 0$

and use CM frame :



$$(p_4 - p_2)^2 - m_c^2 = (p_4 - p_2)^2 = p_4^2 + p_2^2 - 2\, p_2 \cdot p_4$$

$$= 2m^2 - 2\, p_2 \cdot p_4 = 2m^2 - 2\left(E_2 E_4 - |\vec{p}_2||\vec{p}_4| \cos\theta\right)$$

$$= 2m^2 - 2\left(E^2 - |\vec{p}|^2 \cos\theta\right)$$

$$= 2m^2 - 2\left(|\vec{p}|^2 + m^2 - |\vec{p}|^2 \cos\theta\right)$$

Then:

$$(p_4 - p_2)^2 = -2|\vec{p}|^2(1-\cos\theta)$$

Likewise:

$$(p_3 - p_2)^2 = -2|\vec{p}|^2(1+\cos\theta)$$

Thus:

$$M = -\frac{g^2}{|\vec{p}|^2 \sin^2\theta}$$

Since

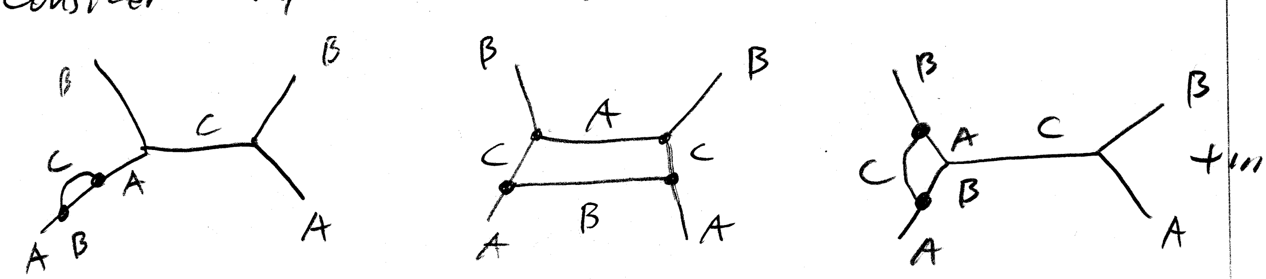
$$M = \frac{g^2(1+\cos\theta)}{-2|\vec{p}|^2(1-\cos\theta)(1+\cos\theta)} + \frac{g^2(1-\cos\theta)}{-2|\vec{p}|^2(1+\cos\theta)(1-\cos\theta)}$$

and

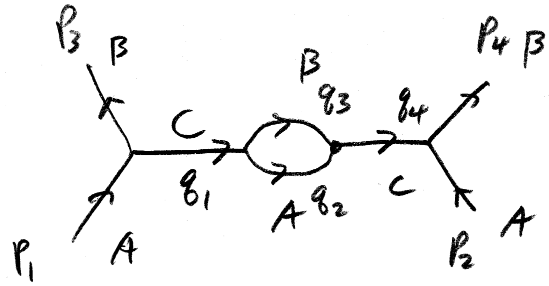
$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{1}{2} \frac{g^4}{(|\vec{p}|^2 \sin^2\theta)^2} \frac{1}{(2E)^2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{\hbar c}{16\pi E |\vec{p}|^2 \sin^2\theta}\right)^2}$$

Consider higher orders:



and



For this last diagram:

$$-iM = (-ig)\ (2\pi)^4\, \delta^4(p_1 - p_3 - q_1)\ \frac{i}{q_1^2 - m_C^2}$$

$$\cdot (-ig)(2\pi)^4\, \delta^4(q_1 - q_3 - q_2)\ \frac{i}{q_3^2 - m_\beta^2}$$

$$\cdot (-ig)\,(2\pi)^4\, \delta^4(q_2 + q_3 - q_4)\ \frac{i}{q_2^2 - m_A^2}$$

$$\cdot (-ig)\ (2\pi)^4\ \delta^4(q_4 + p_2 - p_4)\ \frac{i}{q_4^2 - m_c^2}$$

$$\cdot\ \frac{d^4 q_4}{(2\pi)^4}\ \frac{d^4 q_1}{(2\pi)^4}\ \frac{d^4 q_2}{(2\pi)^4}\ \frac{d^4 q_3}{(2\pi)^4}$$

$$= \frac{(-ig)^4}{(2\pi)^4}\ \frac{1}{[(p_1 - p_3)^2 - m_c^2]} \int \frac{d^4 q}{[(p_1 - p_3 - q)^2 - m_A^2][q^2 - m_B^2]}$$

For large q^2:

$$m \rightarrow \int \frac{q^3 dq}{q^4} \sim \ln q \underset{q\to\infty}{\rightarrow} \infty$$

Try to regularize integral with factor

$$\int [\ldots] \frac{-M^2}{(q^2 - M^2)} = \text{finite} + \underbrace{f(M)}_{\text{divergent}}$$

We can add divergent terms to parameters of theory! Then let

$m_{physical} = m + \delta m(M)$ and

$g_{physical} = g + \delta g(M)$

Then use measured parameters for physical values and systematically ignore divergent terms. This is called <u>renormalization</u>.

Non-divergent terms cause calculable changes in m + g. These are due to vacuum fluctuations. Not all theories are renormalizable but all <u>gauge theories</u> are.