Shajesh presentation

$V(\vec{r})$

$t = -\infty$

$t = +\infty$

$|i, in\rangle$

$|f, out\rangle$

QM energy level

QFT: part states

$$|final\rangle = S\,|i, in\rangle$$

$$T = \langle f, out \,|\, i, in\rangle$$

$$= \langle f, in \,|\, S \,|\, i, in\rangle$$

$$|i, in\rangle = \int \frac{d^3\vec{p}}{(2\pi)^3} \int \frac{dE}{2\pi}\, e^{i(\vec{p}\cdot\vec{x} - Et)}\, \phi(\vec{p}, E)\, \delta(E^2 - |\vec{p}|^2 - m^2)$$

$$\frac{d\sigma}{d\Omega} = |T|^2$$

$$1 \rightarrow 2 + 3$$

phase space

$$d\Gamma' = \frac{2\pi}{\hbar}\left|M(\vec{p}_1, \vec{p}_2, \vec{p}_3)\right|^2 \frac{S}{4\pi E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} (2\pi)^4 \delta^{(4)}(p_1^\mu - p_2^\mu - p_3^\mu)$$

Fermi's "Golden Rule"

Ex. 6.5

CM frame of particle 1
$m_2 = m_3 = 0$
$\vec{p}_1 = 0 \quad , \quad E_1 = m_1$

$$\Gamma = \frac{S}{2\hbar (4\pi)^2} \int d^3 p_2 \int d^3 p_3 \frac{|M|^2}{m_1 \sqrt{|\vec{p}_2|^2 + m_2^2} \sqrt{|\vec{p}_3|^2 + m_3^2}}$$
$$\delta(m_1 - E_2 - E_3)\, \delta^{(3)}(0 - \vec{p}_2 - \vec{p}_3)$$

$$\int_{-\infty}^{\infty} dx\, f(x)\, \delta(x - x_0) = f(x_0)$$

$$\iiint d^3x\, f(\vec{x})\, \delta^3(\vec{x} - \vec{x}_0) = f(\vec{x}_0)$$

$$\Gamma = \frac{S}{2\hbar (4\pi)^2} \int d^3 p_2 \frac{|M|^2 \quad \delta\left(m_1 - \sqrt{p_2^2 + m_2^2} - \sqrt{\vec{p}_2^2 + m_3^2}\right)}{m_1 \sqrt{|\vec{p}_2|^2 + m_2^2} \sqrt{|\vec{p}_2|^2 + m_3^2}}$$

Now use $m_2 = m_3 = 0$

$$\Gamma = \frac{S}{2\hbar (4\pi)^2} \int d^3 p_2 \frac{|M|^2}{m_1 |\vec{p}_2|^2} \delta(m_1 - 2|\vec{p}_2|)$$

$$p_x = |\vec{p}| \sin\theta \cos\phi$$
$$p_y = |\vec{p}| \sin\theta \sin\phi$$
$$p_z = |\vec{p}| \cos\theta$$

$$d^3 \vec{p}_2 = |\vec{p}_2|^2 \sin\theta \, d\theta \, d\phi \, d|\vec{p}_2|$$

$$T' = \frac{S}{2\hbar(4\pi)^2 m_1}\int_0^{\pi}\sin\theta\, d\theta \int_0^{2\pi} d\phi \int_0^{\infty} \frac{|\vec{p}_2|^2\, d|\vec{p}_2|}{|\vec{p}_2|^2}\, |M|^2\, \delta(m_1 - 2|\vec{p}_2|)$$

$$= \frac{S}{2\hbar(4\pi)^2 m_1}\,(4\pi)\int_0^{\infty} d|\vec{p}_2|\, |M|^2\, \delta(m_1 - 2|\vec{p}_2|)$$

$$\int dx\, f(x)\,\delta(x - x_0) = f(x_0)$$

$$\int dx\, f(x)\,\delta(2x - x_0) = \frac{1}{2} f(x_0)$$

$$T' = \frac{S}{4\hbar(4\pi)^2}\,\frac{4\pi}{m_1}\left\{ \left| M(\vec{p}_1 = 0, \vec{p}_2, \vec{p}_3 = -\vec{p}_2) \right|^2 \right\}_{|\vec{p}_2| = \frac{m_1}{2}}$$

$$= \boxed{\frac{S}{16\pi\hbar m}\,|M|^2}$$

For $\pi^0 \to \gamma + \gamma$

$$S = \frac{1}{\gamma!} = \frac{1}{2!} = \frac{1}{2}$$

# Pedro Presentation

## Ex. 6.6 $\quad m_1 \rightarrow m_2 + m_3$

$$T' = \frac{S c^2}{8 \hbar m_1 (2\pi)^2} \int \frac{|M|^2}{E_2 E_3} \delta^4(p_1 - p_2 - p_3)\, d^3|\vec{p}_2|\, d^3|\vec{p}_3|$$

$$E_2 = \sqrt{|\vec{p}_2|^2 + m_2^2}\,, \quad E_3 = \sqrt{|\vec{p}_3|^2 + m_3^2}$$

$$E_1 = m_1$$

$$p_1^\mu = (m_1, 0)$$

$$p_2^\mu = (E_2, \vec{p}_2)$$

$$p_3^\mu = (E_3, \vec{p}_3)$$

$$T' = \frac{S}{8\pi m_1 (2\pi)^2} \int \frac{|M|^2 \delta\left(m_1 - \sqrt{|\vec{p}_2|^2 + m_2^2} - \sqrt{|\vec{p}_3|^2 + m_3^2}\right)}{\sqrt{|\vec{p}_2|^2 + m_2^2}\sqrt{|\vec{p}_3|^2 + m_3^2}}$$

$$\delta^3(0 - \vec{p}_2 - \vec{p}_3)\, d^3|\vec{p}_2|\, d^3|\vec{p}_3|$$

$$= \frac{S}{8\pi m_1 (2\pi)^2} \int \frac{|M|^2 \delta(\quad)}{\sqrt{|\vec{p}_2|^2 + m_2^2}\sqrt{|\vec{p}_2|^2 + m_3^2}}\, d^3|\vec{p}_2|$$

Pedro Ex 6.6 (con't)

$m_1 \to m_2 + m_3$

define $E = \sqrt{p_2^2 + m_2^2} + \sqrt{p_2^2 + m_3^2}$

$$dE = \frac{2\, p_2\, dp_2}{2\sqrt{p_2^2 + m_2^2}} + \frac{2\, p_2\, dp_2}{2\sqrt{p_2^2 + m_3^2}}$$

$$= \left( \frac{\sqrt{p_2^2 + m_3^2} + \sqrt{p_2^2 + m_2^2}}{\sqrt{p_2^2 + m_3^2}\ \sqrt{p_2^2 + m_2^2}} \right) p_2\, dp_2$$

$$\frac{1}{\sqrt{\ }}\,\frac{1}{\sqrt{\ }} = \frac{dE}{E\, p_2\, dp_2}$$

Now

$$T' = \frac{S}{8 m_1 \hbar (2\pi)^2} \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \frac{|M|^2\, \delta(m_1 - E)\, p_2^2\, dp_2 \sin\theta\, d\phi}{\sqrt{p_2^2 + m_2^2}\ \sqrt{p_2^2 + m_3^2}}$$

$$T' = \frac{S\, 4\pi}{8 m_1 \hbar (2\pi)^2} \int_{m_2+m_3}^{\infty} \frac{(M)^2\, \delta(m_1 - E)\, dE}{E\, \cancel{p_2}\, \cancel{dp_2}}\, p_2^{\cancel{2}}\, \cancel{dp_2}$$

$$= \frac{S}{8 m_1 \hbar \pi} \int p_2 |M|^2 \frac{\delta(m_1 - E)\, dE}{E}$$

Use $\int dx\, f(x)\, \delta\!\left(\frac{x}{c} - x_0\right) = c\, f(x_0)$

$$E = \sqrt{p_2^2 + m_2^2} + \sqrt{p_2^2 + m_3^2}$$

$$E = m_1$$

$$m_1^2 = p_2^2 + m_2^2 + p_2^2 + m_3^2 + 2\sqrt{(p_2^2+m_2^2)(p_2^2+m_3^2)}$$

$$\left[m_1^2 - (p_2^2+m_2^2) - (p_2^2+m_3)^2\right]^2 = 4(p_2^2+m_2^2)(p_2^2+m_3^2)$$

$$m_1^4 + p_2^4 + m_2^2 + 2p_2^2 m_2^2 - 2m_1^2(p_2^2+m_2^2) + p_2^4$$

$$+ m_3^4 + 2p_2^2 m_3^2 \underbrace{- 2\left[m_1^2 - (p_2^2+m_2^2)\right](p_2^2+m_3)}_{[\text{expansion}]}$$

$$= 4(p_2^2+m_2^2)(p_2^2+m_3^2)$$

$$[\text{expansion}]:$$

$$-2m_1^2 p_2^2 - 2m_1^2 m_3^2 + 2p_2^4 + 2\cancel{p_2^2} m_3^2 + 2\cancel{m_2^2} p_2^2$$

$$+ 2m_2^2 m_3^2$$

So

$$m_1^4 + m_2^4 + m_3^4 + 2\cancel{p_2^2} m_2 - 2m_1^2 p_2^2 - 2m_1^2 m_2^2$$

$$+ \cancel{4p_2^4} + 2\cancel{p_2^2} m_3^2 + [\text{expansion}]$$

$$= \cancel{4p_2^4} + 4\cancel{p_2^2} m_3^2 + 4p_2^2\cancel{m_2^2} + 4m_2^2 m_3^2 + \cdots$$

$$P_2 = \frac{1}{2m_1}\sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2 m_2^2 - 2m_1^2 m_3^2 - 2m_2^2 m_3^2}$$

$$\boxed{\Gamma' = \frac{S\,|\vec{P}|}{8\pi\hbar\, m_1^2}\,|M|^2}$$

Ye Zhang Scattering :

Ex. 6.7 $\quad 1 + 2 \to 3 + 4$ in CM frame

$$\vec{P}_1 + \vec{P}_2 = 0 \Rightarrow \vec{P}_1 = -\vec{P}_2$$

$$P_1 \cdot P_2 = E_1 E_2 - \vec{P}_1 \cdot \vec{P}_2 = E_1 E_2 + |\vec{P}_1|^2$$

$$E_1^2 = |\vec{P}_1|^2 + m_1^2$$

$$\Rightarrow m_1^2 = E_1^2 - P_1^2 \;, \quad m_2^2 = E_2^2 - P_2$$

Use Golden Rule for Scattering :

$$d\sigma = |M|^2 \frac{\hbar^2 S}{4\sqrt{(P_1 \cdot P_2)^2 - (m_1 m_2)^2}} \left(\frac{d^3 P_3}{(2\pi)^3 2E_3}\right)\left(\frac{d^3 P_4}{(2\pi)^3 2E_4}\right)$$

$$\cdot (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4)$$

$$\sqrt{(p_1\cdot p_2)^2-(m_1m_2)^2} = \sqrt{E_1^2E_2^2 + p_1^4 + 2E_1E_2\,p_1^2 - (E_1^2-p_1^2)(E_2^2-p_2^2)}$$

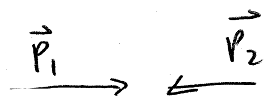
$$= \sqrt{\cancel{E_1^2E_2^2} + \cancel{p_1^4} + 2E_1E_2\,p_1^2 - \cancel{E_1^2E_2^2} - \cancel{p_1^4} + (E_1^2+E_2^2)\,p_1^2}$$

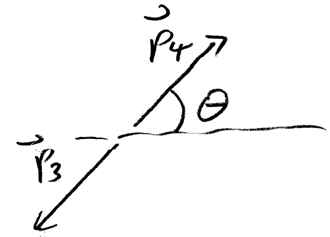
$$= \sqrt{(E_1^2+E_2^2+2E_1E_2)|\vec{p}_1|^2} = (E_1+E_2)|\vec{p}_1|$$

So

$$d\sigma = \frac{|M|^2\,\hbar^2\,S}{4(E_1+E_2)|\vec{p}_1|}\,\frac{d^3p_3\,d^3p_4}{E_3\,E_4\,(2\pi)^6}\,(2\pi)^4\,\delta^4(p_1+p_2-p_3-p_4)$$

$$= \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{(E_1+E_2)|\vec{p}_1|}\,\frac{d^3p_3\,d^3p_4}{E_3E_4}\,\underbrace{\delta^4(p_1+p_2-p_3-p_4)}_{\delta(E_1+E_2-E_3-E_4)\,\delta^3(-\vec{p}_3-\vec{p}_4)}$$




$$d\sigma = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2}{(E_1+E_2)|\vec{p}_1|}\,\frac{\delta\left(E_1+E_2-\sqrt{|\vec{p}_3|^2+m_3^2}-\sqrt{|\vec{p}_4|^2+m_4^2}\right)}{\sqrt{m_3^2+|\vec{p}_3|^2}\,\sqrt{m_4^2+|\vec{p}_4|^2}}$$

$$\delta(-|\vec{p}_3|-|\vec{p}_4|)\,d^3p_3\,d^3p_4$$

Integrate over $d^3(\vec{P}_4)$:

$$d\sigma = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S\,|M|^2}{(E_1+E_2)\,|\vec{P}_1|} \frac{\delta\left(E_1+E_2-\sqrt{|\vec{P}_3|^2+m_3^2}-\sqrt{|\vec{P}_3|^2+m_4^2}\right)}{\sqrt{m_3^2+|\vec{P}_3|^2}\,\sqrt{m_4^2+|\vec{P}_3|^2}}\, d^3|\vec{P}_3|$$

Now $d^3|\vec{P}_3| = |\vec{P}_3|^2\, d|\vec{P}_3| \sin\theta\, d\phi$

$= |\vec{P}_3|^2\, d|\vec{P}_3|\, d\Omega$

So

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S}{(E_1+E_2)\,|\vec{P}_1|} \int_0^\infty |M|^2$$

$$\times \frac{\delta\left(E_1+E_2-\sqrt{m_3^2+|\vec{P}_3|^2}-\sqrt{m_4^2+|\vec{P}_3|^2}\right)|\vec{P}_3|^2\, d|\vec{P}_3|}{\sqrt{m_3^2+|\vec{P}_3|^2}\,\sqrt{m_4^2+|\vec{P}_3|^2}}$$

Use integral result from previous example with $m_1 \to E_1+E_2$ and $m_2 \to m_4$:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S}{(E_1+E_2)\,|\vec{P}_1|} \frac{|\vec{P}_3|}{E_1+E_2} |M|^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S\,|M|^2}{(E_1+E_2)^2} \frac{|\vec{P}_f|}{|\vec{P}_i|}}$$

| expression | | dimension | unit |
|---|---|---|---|
| lifetime | $\tau$ | time | second |
| decay rate | $\Gamma = \frac{1}{\tau}$ | $\frac{1}{\text{time}}$ | $(\text{second})^{-1}$ |
| cross section | $\sigma$ | area $= L^2$ | barn $= 10^{-24}\ \text{cm}^2$ |
| differential cross sec. | $d\sigma/d\Omega$ | " | " |

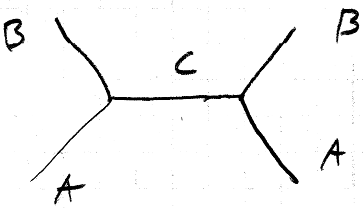
## Zhang presentation (con't)

amplitude $M$ $(mc)^{4-n}$

$n$ = number of external lines

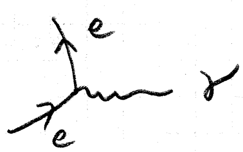
## Sivakuman presentation

Toy theory scattering:

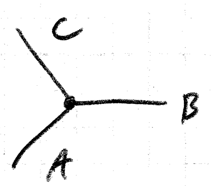



basic diagrams:

for QED:



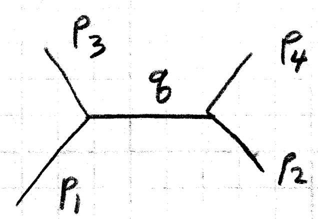

For toy model:




$m_C > m_B > m_A$

For $A + A \to B + B$ :

① Notation:
Label momenta of lines




② $(-ig)$ factor for each vertex

for example: $(-ig)^2$

③ Propagator:

factor of $\dfrac{i}{q_j^2 - m_j^2}$ for each internal line

④ Conservation of Energy and Momentum

write $(2\pi)^4\,\delta^4(k_1 + k_2 + k_3)$ for each vertex

Example: $(2\pi)^4\,\delta^4(p_1 - q - p_3)\,(2\pi)^4\,\delta^4(p_2 + q - p_4)$

⑤ Integrate over internal momenta:

For each internal line write

$$\frac{1}{(2\pi)^4}\,d^4 q_j$$

example:

$$-iM = (-ig)^2\, i \int \frac{(2\pi)^4\,\delta(p_1 - p_3 - q)\;(2\pi)^4\delta(p_2 + q - p_4)}{q^2 - m_c^2}\,\frac{d^4 q}{(2\pi)^4}$$

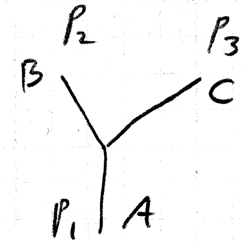
$$= (-ig)^2\,\frac{(2\pi)^4\,\delta(p_1 - p_3 + p_2 - p_4)}{(p_4 - p_2)^2 - m_c^2}$$

⑥ Cancel Delta Function

example: $\boxed{M = \dfrac{g^2}{(p_4 - p_2)^2 - m_c^2}}$

## Section 6.4 :

consider $A \to B + C$




$$-i\mathcal{M} = (-ig)\,\cancel{(2\pi)^4\,\delta^4(p_1 - p_2 - p_3)}$$

$$\mathcal{M} = \boxed{g}$$

From Golden Rule

$$\Gamma = \frac{g^2\,|\vec{p}|}{8\pi\hbar\, m_A^2}$$

$$\tau = \frac{1}{\Gamma} = \frac{8\pi\hbar\, m_A^2}{g^2\,|\vec{p}|}$$

where $|\vec{p}| = \frac{1}{2m_A}\sqrt{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$

[thanks to Pedro!]