

11/14/2002

PHY 5213

Fall 2002

①

Shajesh

presentation

 $t = -\infty$  $|i, in\rangle$ 

QM energy level

QFT: part states

 $V(\vec{r})$ 
 $t = +\infty$  $|f, out\rangle$  $|final\rangle = S |i, in\rangle$ 

$$T = \langle f, out | i, in \rangle$$

$$= \langle f, in | S | i, in \rangle$$

$$|i, in\rangle = \int \frac{d^3\vec{p}}{(2\pi)^3} \int \frac{dE}{2\pi} e^{i(\vec{p}\cdot\vec{x} - Et)} \phi(\vec{p}, E) \delta(E^2 - |\vec{p}|^2 - m^2)$$

$$\frac{d\sigma}{d\Omega} = |T|^2$$

1  $\rightarrow$  2 + 3

$$dT = \frac{2\pi}{\hbar} \left| M(\vec{p}_1, \vec{p}_2, \vec{p}_3) \right|^2 \frac{S}{4\pi E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p_3}{(2\pi)^3 2E_3}$$

$$(2\pi)^4 \delta^{(4)}(p_1 - p_2 - p_3)$$

Fermi's "Golden Rule"

Ex. 6.5

CM frame of particle 1  
m<sub>2</sub> = m<sub>3</sub> = 0  
P<sub>1</sub> = 0, E<sub>1</sub> = m<sub>1</sub>

$$T = \frac{S}{2\hbar(4\pi)^2} \int d^3 p_2 \int d^3 p_3 \frac{|M|^2}{m_1 \sqrt{|\vec{p}_2|^2 + m_2^2} \sqrt{|\vec{p}_3|^2 + m_3^2}}$$

$$\delta(m_1 - E_2 - E_3) \delta^{(3)}(0 - \vec{p}_2 - \vec{p}_3)$$

$$\int_{-\infty}^{\infty} dx f(x) \delta(x - x_0) = f(x_0)$$

$$\iiint d^3 x f(\vec{x}) \delta^3(\vec{x} - \vec{x}_0) = f(\vec{x}_0)$$

$$T' = \frac{S}{2\hbar(4\pi)^2} \int d^3 p_2 \frac{|M|^2}{m_1 \sqrt{|\vec{p}_2|^2 + m_2^2} \sqrt{|\vec{p}_2|^2 + m_3^2}} \delta(m_1 - \sqrt{p_2^2 + m_2^2} - \sqrt{p_2^2 + m_3^2})$$

Now use m<sub>2</sub> = m<sub>3</sub> = 0

$$T' = \frac{S}{2\hbar(4\pi)^2} \int d^3 p_2 \frac{|M|^2}{m_1 |\vec{p}_2|^2} \delta(m_1 - 2|\vec{p}_2|)$$

$$p_x = |\vec{p}| \sin \theta \cos \phi$$

$$p_y = |\vec{p}| \sin \theta \sin \phi$$

$$p_z = |\vec{p}| \cos \theta$$

$$d^3(\vec{p}) = |\vec{p}|^2 \sin \theta d\theta d\phi d|\vec{p}|$$

100 SHEETS FULLER 8 SQUARE  
42-331 50 SHEETS EVASOFT 8 SQUARE  
42-382 100 SHEETS EVASOFT 8 SQUARE  
42-383 100 SHEETS EVASOFT 8 SQUARE  
42-384 100 RECYCLED WHITE 8 SQUARE  
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42-400 200 RECYCLED WHITE 8 SQUARE  
National Brand  
Made in U.S.A.

(3)

$$T' = \frac{S}{2 \hbar (4\pi)^2 m_1} \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \int_0^\infty \frac{|\vec{p}_2|^2 d|\vec{p}_2|}{|\vec{p}_2|^2} |M|^2 \delta(m_1 - 2|\vec{p}_2|)$$

$$= \frac{S}{2 \hbar (4\pi)^2 m_1} (4\pi) \int_0^\infty d|\vec{p}_2| |M|^2 \delta(m_1 - 2|\vec{p}_2|)$$

$$\int dx f(x) \delta(x - x_0) = f(x_0)$$

$$\int dx f(x) \delta(2x - x_0) = \frac{1}{2} f(x_0)$$

$$T' = \frac{S}{4 \hbar (4\pi)^2} \frac{4\pi}{m_1} \left\{ |M(\vec{p}_1=0, \vec{p}_2, \vec{p}_3=-\vec{p}_2)|^2 \right\}_{|\vec{p}_2| = \frac{m_1}{2}}$$

$$= \boxed{\frac{S}{16\pi \hbar m} |M|^2}$$

For  $\pi^0 \rightarrow \gamma + \gamma$

$$S = \frac{1}{2!} = \frac{1}{2!} = \frac{1}{2}$$

# Pedro presentation

Ex. 6.6  $m_1 \rightarrow m_2 + m_3$

$$T^1 = \frac{S c^2}{8\pi m_1 (2\pi)^2} \int \frac{|M|^2}{E_2 E_3} \delta^4(-P_1 - P_2 - P_3) d^3\vec{P}_2 d^3\vec{P}_3$$

$$E_2 = \sqrt{|\vec{P}_2|^2 + m_2^2}, \quad E_3 = \sqrt{|\vec{P}_3|^2 + m_3^2}$$

$$E_1 = m_1$$

$$P_1^\mu = (m_1, 0)$$

$$P_2^\mu = (E_2, \vec{P}_2)$$

$$P_3^\mu = (E_3, \vec{P}_3)$$

$$T^1 = \frac{S}{8\pi m_1 (2\pi)^2} \int \frac{|M|^2 \delta(m_1 - \sqrt{|\vec{P}_2|^2 + m_2^2} - \sqrt{|\vec{P}_3|^2 + m_3^2})}{\sqrt{|\vec{P}_2|^2 + m_2^2} \sqrt{|\vec{P}_3|^2 + m_3^2}} \delta^3(0 - \vec{P}_2 - \vec{P}_3) d^3\vec{P}_2 d^3\vec{P}_3$$

$$= \frac{S}{8\pi m_1 (2\pi)^2} \int \frac{|M|^2 \delta(\quad)}{\sqrt{|\vec{P}_2|^2 + m_2^2} \sqrt{|\vec{P}_2|^2 + m_3^2}} d^3\vec{P}_2$$

18782 500 SHEETS FULLER 9 SQUARE  
 40382 500 SHEETS FULLER 9 SQUARE  
 42382 100 SHEETS FULLER 9 SQUARE  
 48389 200 SHEETS FULLER 9 SQUARE  
 49389 200 SHEETS FULLER 9 SQUARE  
 49389 200 RECYCLED WHITE 9 SQUARE  
 Made in U.S.A.



## Pedro Ex 6.6 (cont)

$$m_1 \rightarrow m_2 + m_3$$

$$\text{define } E = \sqrt{p_2^2 + m_2^2} + \sqrt{p_2^2 + m_3^2}$$

$$dE = \frac{2 p_2 dp_2}{2 \sqrt{p_2^2 + m_2^2}} + \frac{2 p_2 dp_2}{2 \sqrt{p_2^2 + m_3^2}}$$

$$= \left( \frac{\sqrt{p_2^2 + m_3^2} + \sqrt{p_2^2 + m_2^2}}{\sqrt{p_2^2 + m_3^2} \sqrt{p_2^2 + m_2^2}} \right) p_2 dp_2$$

$$\frac{1}{\sqrt{}} \frac{1}{\sqrt{}} = \frac{dE}{E p_2 dp_2}$$

Now

$$T_1 = \frac{S}{8 m_1 \hbar (2\pi)^2} \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \frac{|M|^2 \delta(m_1 - E) p_2^2 dp_2 \sin\theta d\theta}{\sqrt{p_2^2 + m_2^2} \sqrt{p_2^2 + m_3^2}}$$

$$T_1 = \frac{S}{8 m_1 \hbar (2\pi)^2} \int_{m_2 + m_3}^{\infty} \frac{|M|^2 \delta(m_1 - E) dE p_2^2 dp_2}{E p_2 dp_2}$$

$$= \frac{S}{8 m_1 \hbar \pi} \int p_2 |M|^2 \frac{\delta(m_1 - E) dE}{E}$$

$$\text{Use } \int dx f(x) \delta\left(\frac{x}{c} - x_0\right) = c f(x_0)$$

$$E = \sqrt{p_2^2 + m_2^2} + \sqrt{p_2^2 + m_3^2}$$

$$E = m_1$$

$$m_1^2 = p_2^2 + m_2^2 + p_2^2 + m_3^2 + 2 \sqrt{(p_2^2 + m_2^2)(p_2^2 + m_3^2)}$$

$$\left[ m_1^2 - (p_2^2 + m_2^2) - (p_2^2 + m_3^2) \right]^2 = 4 (p_2^2 + m_2^2)(p_2^2 + m_3^2)$$

$$m_1^4 + p_2^4 + m_2^2 + 2p_2^2 m_2^2 - 2m_1^2 (p_2^2 + m_2^2) + p_2^4$$

$$+ m_3^4 + 2p_2^2 m_3^2 - 2 \left[ m_1^2 - (p_2^2 + m_2^2) \right] (p_2^2 + m_3^2)$$

$$= 4 (p_2^2 + m_2^2)(p_2^2 + m_3^2)$$

(expansion):

$$- 2m_1^2 p_2^2 - 2m_1^2 m_3^2 + 2p_2^4 + 2p_2^2/m_3^2 + 2m_3^2/p_2^2 + 2m_2^2 m_3^2$$

So

$$m_1^4 + m_2^4 + m_3^4 + 2p_2^2/m_2 - 2m_1^2 p_2^2 - 2m_1^2 m_2^2$$

$$+ 4p_2^4 + 2p_2^2/m_3^2 + [\text{expansion}]$$

$$= 4p_2^4 + 4p_2^2/m_3^2 + 4p_2^2/m_2^2 + 4m_2^2 m_3^2 + \dots$$

$$P_2 = \frac{1}{2m_1} \sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2 m_2^2 - 2m_1^2 m_3^2 - 2m_2^2 m_3^2}$$

$$T^1 = \frac{S |\vec{P}_1|}{8\pi \hbar m_1^2} |M|^2$$

Ye Zhang Scattering :

Ex. 6.7  $1 + 2 \rightarrow 3 + 4$  in CM frame

$$\vec{P}_1 + \vec{P}_2 = 0 \Rightarrow \vec{P}_1 = -\vec{P}_2$$

$$P_1 \cdot P_2 = E_1 E_2 - \vec{P}_1 \cdot \vec{P}_2 = E_1 E_2 + |\vec{P}_1|^2$$

$$E_1^2 = |\vec{P}_1|^2 + m_1^2$$

$$\Rightarrow m_1^2 = E_1^2 - P_1^2, \quad m_2^2 = E_2^2 - P_2^2$$

Use Golden Rule for Scattering :

$$d\sigma = |M|^2 \frac{S}{4 \sqrt{(P_1 \cdot P_2)^2 - (m_1 m_2)^2}} \left( \frac{d^3 p_3}{(2\pi)^3 2E_3} \right) \left( \frac{d^3 p_4}{(2\pi)^3 2E_4} \right)$$

$$\cdot (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4)$$

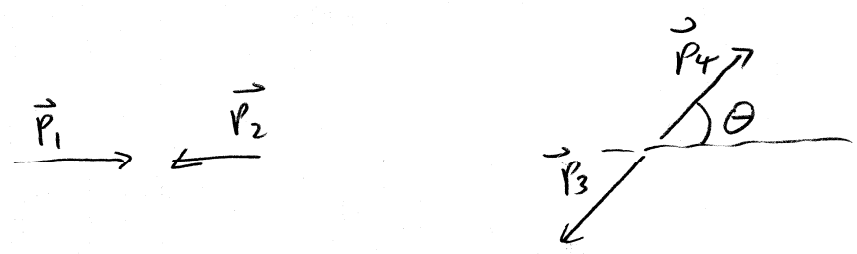
50 SHEETS PER 8 SQUARE  
 50 SHEETS PER 8 SQUARE  
 100 SHEETS PER 8 SQUARE  
 200 SHEETS PER 8 SQUARE  
 400 SHEETS PER 8 SQUARE  
 200 RECYCLED WHITE  
 Made in U.S.A.

$$\begin{aligned} \sqrt{(P_1 \cdot P_2)^2 - (m_1 m_2)^2} &= \sqrt{E_1^2 E_2^2 + P_1^4 + 2E_1 E_2 P_1^2 - (E_1^2 - P_1^2)(E_2^2 - P_2^2)} \\ &= \sqrt{E_1^2/E_2^2 + P_1^4 + 2E_1 E_2 P_1^2 - E_2^2/E_2^2 - P_1^4 + (E_1^2 + E_2^2) P_1^2} \\ &= \sqrt{(E_1^2 + E_2^2 + 2E_1 E_2) |\vec{P}_1|^2} = (E_1 + E_2) |\vec{P}_1| \end{aligned}$$

So

$$d\sigma = \frac{|m|^2 \hbar^2 S}{4(E_1 + E_2) |\vec{P}_1|} \frac{d^3 P_3 d^3 P_4}{E_3 E_4 (2\pi)^6} (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4)$$

$$= \left(\frac{\hbar}{8\pi}\right)^2 \frac{S |M|^2}{(E_1 + E_2) |\vec{P}_1|} \frac{d^3 P_3 d^3 P_4}{E_3 E_4} \underbrace{\delta^4(P_1 + P_2 - P_3 - P_4)}_{\delta(E_1 + E_2 - E_3 - E_4) \delta^3(-\vec{P}_3 - \vec{P}_4)}$$



$$d\sigma = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S |M|^2}{(E_1 + E_2) |\vec{P}_1|} \frac{\delta(E_1 + E_2 - \sqrt{|\vec{P}_3|^2 + m_3^2} - \sqrt{|\vec{P}_4|^2 + m_4^2})}{\sqrt{m_3^2 + |\vec{P}_3|^2} \sqrt{m_4^2 + |\vec{P}_4|^2}} \delta^3(-\vec{P}_3 - \vec{P}_4) d^3 P_3 d^3 P_4$$

600 SHEETS FULLER 8 SQUARE  
42 384  
60 SHEETS REFERENCE SQUARE  
42 362  
100 SHEETS EYE-GLASS SQUARE  
42 389  
100 RECYCLED WHITE 8 SQUARE  
42 393  
200 RECYCLED WHITE 8 SQUARE  
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11/21/2001

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Integrate over  $d^3(\vec{p}_4)$ :

$$d\sigma = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S |M|^2}{(E_1 + E_2) |\vec{p}_1|} \frac{\delta(E_1 + E_2 - \sqrt{|\vec{p}_3|^2 + m_3^2} - \sqrt{|\vec{p}_4|^2 + m_4^2})}{\sqrt{m_3^2 + |\vec{p}_3|^2} \sqrt{m_4^2 + |\vec{p}_3|^2}} d^3|\vec{p}_3|$$

Now  $d^3|\vec{p}_3| = |\vec{p}_3|^2 d|\vec{p}_3| \sin\theta d\phi$

$$= |\vec{p}_3|^2 d|\vec{p}_3| d\Omega$$

So

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S}{(E_1 + E_2) |\vec{p}_1|} \int_0^\infty |M|^2$$

$$\times \frac{\delta(E_1 + E_2 - \sqrt{m_3^2 + |\vec{p}_3|^2} - \sqrt{m_4^2 + |\vec{p}_3|^2}) |\vec{p}_3|^2 d|\vec{p}_3|}{\sqrt{m_3^2 + |\vec{p}_3|^2} \sqrt{m_4^2 + |\vec{p}_3|^2}}$$

Use integral result from previous example  
with  $m_1 \rightarrow E_1 + E_2$  and  $m_2 \rightarrow m_4$ :

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S}{(E_1 + E_2) |\vec{p}_1|} \frac{|\vec{p}_3|}{E_1 + E_2} |M|^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S |M|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_4|}{|\vec{p}_1|}}$$

	expression	dimension	unit
lifetime	$\tau$	time	second
decay rate	$\Gamma = \frac{1}{\tau}$	$\frac{1}{\text{time}}$	(second) <sup>-1</sup>
Cross section	$\sigma$	area = L <sup>2</sup>	barn = 10 <sup>-24</sup> cm <sup>2</sup>
Differential cross sec.	$d\sigma/d\Omega$	"	"

Zhang presentation (cont)

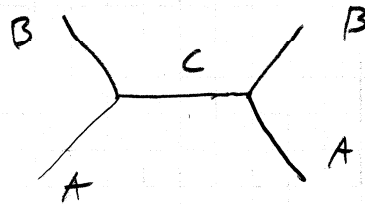
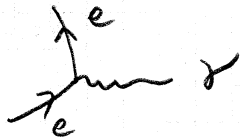
amplitude  $M \quad (m_c)^{4-n}$

$n =$  number of external lines

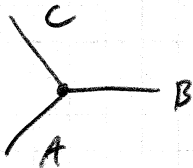
Siva kuman presentation

Toy theory scattering:

basic diagrams;  
for QED:



For toy model:

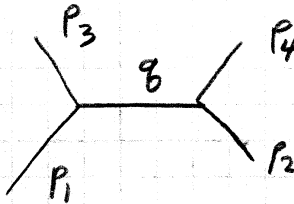


$m_c > m_B > m_A$

For  $A + A \rightarrow B + B$ :

① Notation:

Label momenta  
of lines



②  $(-ig)$  factor for each vertex

for example:  $(-ig)^2$

(3) Propagator :  
factor of  $\frac{i}{q_j^2 - m_j^2}$  for each internal line

(4) Conservation of Energy and Momentum  
write  $(2\pi)^4 \delta^4(l_1 + l_2 + l_3)$  for each vertex

Example :  $(2\pi)^4 \delta^4(p_1 - q - p_3) (2\pi)^4 \delta^4(p_2 + q - p_4)$

(5) Integrate over internal momenta :

For each internal line write

$$\frac{1}{(2\pi)^4} d^4 q_j$$

example :

$$-iM = (-ig)^2 i \int \frac{(2\pi)^4 \delta^4(p_1 - p_3 - q) (2\pi)^4 \delta^4(p_2 + q - p_4) d^4 q}{q^2 - m_c^2} (2\pi)^4$$

$$= (-ig)^2 \frac{(2\pi)^4 \delta^4(p_1 - p_3 + p_2 - p_4)}{(p_4 - p_2)^2 - m_c^2}$$

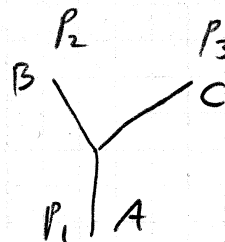
(6) Cancel Delta Function

example :

$$M = \frac{g^2}{(p_4 - p_2)^2 - m_c^2}$$

Section 6.4 :

Consider



~~$$-im = (-ig) (2\pi)^4 \delta^4(p_1 - p_2 - p_3)$$~~

$$m = \boxed{g}$$

From Golden Rule

$$\Gamma = \frac{g^2 |\vec{P}|}{8\pi^4 m_A^2}$$

$$\Gamma = \frac{1}{\Gamma} = \frac{8\pi^4 m_A^2}{g^2 |\vec{P}|}$$

where  $|\vec{P}| = \frac{1}{2m_A} \sqrt{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$

[thanks to Pedro!]