Casimir effect between poor conductors: parallel plates

Simen Ellingsen
NTNU
Trondheim, NO

March 14, 2008

Outline

Temperature anomaly for semiconductors

Reported by Geyer, Klimchitskaya, Mostepanenko (Phys. Rev. D, **72**, 085009)

Temperature anomaly for semiconductors

Reported by Geyer, Klimchitskaya, Mostepanenko
(Phys. Rev. D, **72**, 085009)
Permittivity of a conducting material in *local* formalism (Drude):

Temperature anomaly for semiconductors

Reported by Geyer, Klimchitskaya, Mostepanenko
(Phys. Rev. D, **72**, 085009)
Permittivity of a conducting material in *local* formalism (Drude):

$$\varepsilon(i\zeta) = 1 + \frac{\varepsilon_\infty - 1}{1 + \zeta^2/\omega_0^2} + \frac{4\pi\sigma}{\zeta}$$

$\varepsilon_\infty, \omega_0$ material parameters. σ: conductivity. ζ is imaginary frequency: $i\omega = \zeta$.

Temperature anomaly for semiconductors

Reported by Geyer, Klimchitskaya, Mostepanenko
(Phys. Rev. D, **72**, 085009)
Permittivity of a conducting material in *local* formalism (Drude):

$$\varepsilon(i\zeta) = 1 + \frac{\varepsilon_\infty - 1}{1 + \zeta^2/\omega_0^2} + \frac{4\pi\sigma}{\zeta}$$

$\varepsilon_\infty, \omega_0$ material parameters. σ: conductivity. ζ is imaginary frequency: $i\omega = \zeta$.

- $\sigma > 0$ (however small): $\lim_{\zeta\to 0} \varepsilon = \infty$;

Temperature anomaly for semiconductors

Reported by Geyer, Klimchitskaya, Mostepanenko
(Phys. Rev. D, **72**, 085009)
Permittivity of a conducting material in *local* formalism (Drude):

$$\varepsilon(i\zeta) = 1 + \frac{\varepsilon_\infty - 1}{1 + \zeta^2/\omega_0^2} + \frac{4\pi\sigma}{\zeta}$$

$\varepsilon_\infty, \omega_0$ material parameters. σ: conductivity. ζ is imaginary frequency: $i\omega = \zeta$.

- $\sigma > 0$ (however small): $\lim_{\zeta\to 0} \varepsilon = \infty$;
- $\sigma = 0$: $\lim_{\zeta\to 0} \varepsilon = \varepsilon_\infty$;

Temperature anomaly for semiconductors

Transverse magnetic (TM) reflection coefficient between vacuum and material characterised by $\varepsilon(i\zeta)$:

Temperature anomaly for semiconductors

Transverse magnetic (TM) reflection coefficient between vacuum and material characterised by $\varepsilon(i\zeta)$:

$$r_{\mathrm{TM}} = \frac{\varepsilon\kappa - \sqrt{\kappa^2 + \zeta^2(\varepsilon - 1)}}{\varepsilon\kappa + \sqrt{\kappa^2 + \zeta^2(\varepsilon - 1)}};$$

- $\kappa^2 = k_\perp^2 + \zeta^2 = -k_z^2$. ($\hat{z}$ normal to surface)
- $k_\perp$: wave vector $\|$ surface

Temperature anomaly for semiconductors

Transverse magnetic (TM) reflection coefficient between vacuum and material characterised by $\varepsilon(i\zeta)$:

$$r_{\mathrm{TM}} = \frac{\varepsilon\kappa - \sqrt{\kappa^2 + \zeta^2(\varepsilon - 1)}}{\varepsilon\kappa + \sqrt{\kappa^2 + \zeta^2(\varepsilon - 1)}};$$

- $\kappa^2 = k_\perp^2 + \zeta^2 = -k_z^2$. ($\hat{z}$ normal to surface)
- $k_\perp$: wave vector $\|$ surface

Thus:

- $\sigma > 0$ (however small): $\lim_{\zeta \to 0} r_{\mathrm{TM}} = 1$;

Temperature anomaly for semiconductors

Transverse magnetic (TM) reflection coefficient between vacuum and material characterised by $\varepsilon(i\zeta)$:

$$r_{\mathrm{TM}} = \frac{\varepsilon\kappa - \sqrt{\kappa^2 + \zeta^2(\varepsilon - 1)}}{\varepsilon\kappa + \sqrt{\kappa^2 + \zeta^2(\varepsilon - 1)}};$$

- $\kappa^2 = k_\perp^2 + \zeta^2 = -k_z^2$. ($\hat{z}$ normal to surface)
- $k_\perp$: wave vector $\|$ surface

Thus:

- $\sigma > 0$ (however small): $\lim_{\zeta \to 0} r_{\mathrm{TM}} = 1$;
- $\sigma = 0$: $\lim_{\zeta \to 0} r_{\mathrm{TM}} = (\varepsilon_\infty - 1)/(\varepsilon_\infty + 1) < 1$;

Temperature anomaly for semiconductors

For most semiconductors at room temperature the following holds:

$$\zeta_1 = 2\pi T \gg 4\pi\sigma > 0 \qquad (1)$$

Temperature anomaly for semiconductors

For most semiconductors at room temperature the following holds:

$$\zeta_1 = 2\pi T \gg 4\pi\sigma > 0 \tag{1}$$

- It follows then that:

$$r_{\mathrm{TM}}(0,\kappa) = 1; \quad r_{\mathrm{TM}}(i\zeta_1,\kappa) = (\varepsilon_\infty - 1)/(\varepsilon_\infty + 1);$$

Temperature anomaly for semiconductors

For most semiconductors at room temperature the following holds:

$$\zeta_1 = 2\pi T \gg 4\pi\sigma > 0 \tag{1}$$

- It follows then that:

$$r_{\mathrm{TM}}(0,\kappa) = 1; \quad r_{\mathrm{TM}}(i\zeta_1,\kappa) = (\varepsilon_\infty - 1)/(\varepsilon_\infty + 1);$$

- If $\sigma(T) \to 0$ linearly or faster as $T \to 0$, (1) holds for all sub-room temperatures as well.

Temperature anomaly for semiconductors

For most semiconductors at room temperature the following holds:

$$\zeta_1 = 2\pi T \gg 4\pi\sigma > 0 \qquad (1)$$

- It follows then that:

$$r_{\mathrm{TM}}(0,\kappa) = 1; \quad r_{\mathrm{TM}}(i\zeta_1,\kappa) = (\varepsilon_\infty - 1)/(\varepsilon_\infty + 1);$$

- If $\sigma(T) \to 0$ linearly or faster as $T \to 0$, (1) holds for all sub-room temperatures as well.
- As $T \to 0$, r_{TM} becomes truly discontinuous as $\zeta_1 \to 0$.

Temperature anomaly for semiconductors

Taken at face value this is problematic...

Temperature anomaly for semiconductors

Taken at face value this is problematic...

- (Helmholz) free energy at finite T (Lifshitz 1956), TM mode:

$$\mathcal{F} = \frac{T}{2\pi} \sum_{m=0}^{\infty}{}' \int_{\zeta_m}^{\infty} d\kappa \kappa \ln(1 - r_{\mathrm{TM}}^2 e^{-2\kappa a})$$

where $\zeta_m = 2\pi m T; \quad m \in \mathbb{N}$.

Temperature anomaly for semiconductors

Taken at face value this is problematic...

- (Helmholz) free energy at finite T (Lifshitz 1956), TM mode:

$$\mathcal{F} = \frac{T}{2\pi} \sum_{m=0}^{\infty}{}' \int_{\zeta_m}^{\infty} d\kappa\kappa \ln(1 - r_{\mathrm{TM}}^2 e^{-2\kappa a})$$

  where $\zeta_m = 2\pi m T; \quad m \in \mathbb{N}$.

- As $T \to 0$, sum becomes Riemann integral

$$\sum_{m=0}^{\infty}{}' \to \int_0^{\infty} dm$$

Temperature anomaly for semiconductors

- Temperature correction is difference between sum and integral, given by Euler-Maclaurin formula.

Temperature anomaly for semiconductors

- Temperature correction is difference between sum and integral, given by Euler-Maclaurin formula.
- Requires summand to be continuous...

Temperature anomaly for semiconductors

- Temperature correction is difference between sum and integral, given by Euler-Maclaurin formula.
- Requires summand to be continuous...
- ...so must separate out discontinuous addition: difference between $m = 0$ terms with $r_{\mathrm{TM}} = 1$ and $r_{\mathrm{TM}} = (\varepsilon - 1)/(\varepsilon + 1)$.

Temperature anomaly for semiconductors

- Discontinuity gives additional correction term:

$$\mathcal{F} \sim \frac{T}{4\pi}\left\{\int_0^\infty d\kappa\kappa \ln \frac{1-(\frac{\varepsilon_\infty-1}{\varepsilon_\infty+1})^2 e^{-2\kappa a}}{1-e^{-2\kappa a}}\right\}$$

$$= \frac{T}{16\pi a^3}[\mathrm{Li}_3\left(\frac{(\varepsilon_\infty-1)^2}{(\varepsilon_\infty+1)^2}\right)-\zeta(3)]$$

Temperature anomaly for semiconductors

- Discontinuity gives additional correction term:

$$\mathcal{F} \sim \frac{T}{4\pi}\left\{\int_0^\infty d\kappa\kappa \ln \frac{1-(\frac{\varepsilon_\infty-1}{\varepsilon_\infty+1})^2 e^{-2\kappa a}}{1-e^{-2\kappa a}}\right\}$$
$$= \frac{T}{16\pi a^3}[\mathrm{Li}_3\left(\frac{(\varepsilon_\infty-1)^2}{(\varepsilon_\infty+1)^2}\right)-\zeta(3)]$$

- Entropy contribution at $T=0$:

$$S = -\left(\frac{\partial\mathcal{F}}{\partial\mathcal{T}}\right)_V = \text{const.} > 0$$

Temperature anomaly for semiconductors

- Discontinuity gives additional correction term:

$$\mathcal{F} \sim \frac{T}{4\pi}\left\{\int_0^\infty d\kappa\kappa \ln \frac{1-(\frac{\varepsilon_\infty-1}{\varepsilon_\infty+1})^2 e^{-2\kappa a}}{1-e^{-2\kappa a}}\right\}$$
$$= \frac{T}{16\pi a^3}[\mathrm{Li}_3\left(\frac{(\varepsilon_\infty-1)^2}{(\varepsilon_\infty+1)^2}\right)-\zeta(3)]$$

- Entropy contribution at $T=0$:

$$S = -\left(\frac{\partial\mathcal{F}}{\partial\mathcal{T}}\right)_V = \text{const.} > 0$$

- $\Rightarrow$ Violation of 3rd law of thermodynamics (Nernst's theorem)!

Temperature anomaly for semiconductors

Mostepanenko et al.'s solution: ignore the conductivity

Temperature anomaly for semiconductors

Mostepanenko et al.'s solution: ignore the conductivity

$$\varepsilon(i\zeta) = 1 + \frac{\varepsilon_\infty - 1}{1 + \zeta^2/\omega_0^2} + \cancel{\frac{4\pi\sigma}{\zeta}}$$

Outline

Physical consideration

What does all this mean physically?

Physical consideration

What does all this mean physically?

- TM at $\zeta = 0$ corresponds to an electrostatic field.

Physical consideration

What does all this mean physically?

- TM at $\zeta = 0$ corresponds to an electrostatic field.
- So $r_{\text{TM}}(\zeta = 0) = 1$ corresponds to perfect screening of electostatic field from interior of material by free charges.

Physical consideration

What does all this mean physically?

- TM at $\zeta = 0$ corresponds to an electrostatic field.
- So $r_{\text{TM}}(\zeta = 0) = 1$ corresponds to perfect screening of electostatic field from interior of material by free charges.
- When conductivity decreases, electrons can no longer perfect the screening
  $\Rightarrow r_{\text{TM}}(\zeta = 0)$ should drop below 1 at some point.

Physical consideration

What does all this mean physically?

- TM at $\zeta = 0$ corresponds to an electrostatic field.
- So $r_{\mathrm{TM}}(\zeta = 0) = 1$ corresponds to perfect screening of electostatic field from interior of material by free charges.
- When conductivity decreases, electrons can no longer perfect the screening
  $\Rightarrow r_{\mathrm{TM}}(\zeta = 0)$ should drop below 1 at some point.
- If so, the form $\varepsilon(i\zeta) \sim 4\pi\sigma/\zeta$ behaviour breaks down for very small σ.

Physical consideration

What does all this mean physically?

- TM at $\zeta = 0$ corresponds to an electrostatic field.
- So $r_{\mathrm{TM}}(\zeta = 0) = 1$ corresponds to perfect screening of electostatic field from interior of material by free charges.
- When conductivity decreases, electrons can no longer perfect the screening
  $\Rightarrow r_{\mathrm{TM}}(\zeta = 0)$ should drop below 1 at some point.
- If so, the form $\varepsilon(i\zeta) \sim 4\pi\sigma/\zeta$ behaviour breaks down for very small σ.
- Intuitively: ε should approach ε_∞ smoothly.

Physical consideration

What does all this mean physically?

- TM at $\zeta = 0$ corresponds to an electrostatic field.
- So $r_{\mathrm{TM}}(\zeta = 0) = 1$ corresponds to perfect screening of electostatic field from interior of material by free charges.
- When conductivity decreases, electrons can no longer perfect the screening
  $\Rightarrow r_{\mathrm{TM}}(\zeta = 0)$ should drop below 1 at some point.
- If so, the form $\varepsilon(i\zeta) \sim 4\pi\sigma/\zeta$ behaviour breaks down for very small σ.
- Intuitively: ε should approach ε_∞ smoothly.
- Intuitively: Infinitesimal conductivity cannot give rise to large correction.

Physical consideration

Alternative?

- Mostepanenko et.al.ś solution also unsatisfactory: ignores a physical effect which is evidently present.

My conclusion: one should look for intermediate solutions.

Outline

Spatial dispersion

- Assume free electrons behave like an electron-hole plasma.

Spatial dispersion

- Assume free electrons behave like an electron-hole plasma.
- Response to external field: Debye-Hückel screening.

Spatial dispersion

- Assume free electrons behave like an electron-hole plasma.
- Response to external field: Debye-Hückel screening.
- Introduce longitudinal permittivity:

$$\varepsilon_L = \bar{\varepsilon}(i\zeta)(1 + (k\lambda)^{-2}).$$

$k = |\mathbf{k}|$; λ is Debye-Hückel screening length:

$$\lambda = \sqrt{\frac{\bar{\varepsilon} T}{4\pi e^2 (n_e + n_h)}}$$

e: elemetary charge, n_e, n_h: number density of electrons, holes. $\bar{\varepsilon}$ is permittivity without conductivity term.

Spatial dispersion

- **k**-dependent permittivity:

$$\varepsilon_{ij} = \varepsilon_L \frac{k_i k_j}{\mathbf{k}^2} + \bar{\varepsilon}(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2})$$

Spatial dispersion

- **k**-dependent permittivity:

$$\varepsilon_{ij} = \varepsilon_L \frac{k_i k_j}{\mathbf{k}^2} + \bar{\varepsilon}(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2})$$

- Solving Maxwell's equations on both side of interface, joining solutions (Kliewer & Fuchs 1968).

Spatial dispersion

- **k**-dependent permittivity:

$$\varepsilon_{ij} = \varepsilon_L \frac{k_i k_j}{\mathbf{k}^2} + \bar{\varepsilon}(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2})$$

- Solving Maxwell's equations on both side of interface, joining solutions (Kliewer & Fuchs 1968).
- Gives TM surface impedance. Inserted into reflection coefficient.

Spatial dispersion

- **k**-dependent permittivity:

$$\varepsilon_{ij} = \varepsilon_L \frac{k_i k_j}{\mathbf{k}^2} + \bar{\varepsilon}(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2})$$

- Solving Maxwell's equations on both side of interface, joining solutions (Kliewer & Fuchs 1968).
- Gives TM surface impedance. Inserted into reflection coefficient.
- (TE mode remains unchanged)

Spatial dispersion

- Result:

$$r_{\mathrm{TM}} = \frac{\bar{\varepsilon}\kappa\sqrt{1+(k_\perp\lambda)^{-2}} - \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}}{\bar{\varepsilon}\kappa\sqrt{1+(k_\perp\lambda)^{-2}} + \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}};$$

Spatial dispersion

- Result:

$$r_{\mathrm{TM}} = \frac{\bar{\varepsilon}\kappa\sqrt{1+(k_\perp\lambda)^{-2}} - \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}}{\bar{\varepsilon}\kappa\sqrt{1+(k_\perp\lambda)^{-2}} + \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}};$$

- For $\sigma \to 0$: goes smoothly to

$$r_{\mathrm{TM}} = \frac{\bar{\varepsilon}\kappa - \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}}{\bar{\varepsilon}\kappa + \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}};$$

Spatial dispersion

- Result:

$$r_{\mathrm{TM}} = \frac{\bar{\varepsilon}\kappa\sqrt{1+(k_\perp\lambda)^{-2}} - \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}}{\bar{\varepsilon}\kappa\sqrt{1+(k_\perp\lambda)^{-2}} + \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}};$$

- For $\sigma \to 0$: goes smoothly to

$$r_{\mathrm{TM}} = \frac{\bar{\varepsilon}\kappa - \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}}{\bar{\varepsilon}\kappa + \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}};$$

- All terms $m > 1$ must remain unchanged $\Rightarrow$ theory applies only for $2\pi T \gg 4\pi\sigma$.

Spatial dispersion

- Result:

$$r_{\mathrm{TM}} = \frac{\bar{\varepsilon}\kappa\sqrt{1+(k_\perp\lambda)^{-2}} - \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}}{\bar{\varepsilon}\kappa\sqrt{1+(k_\perp\lambda)^{-2}} + \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}};$$

- For $\sigma \to 0$: goes smoothly to

$$r_{\mathrm{TM}} = \frac{\bar{\varepsilon}\kappa - \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}}{\bar{\varepsilon}\kappa + \sqrt{\kappa^2+\zeta^2(\bar{\varepsilon}-1)}};$$

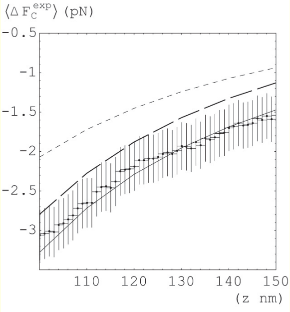
- All terms $m > 1$ must remain unchanged $\Rightarrow$ theory applies only for $2\pi T \gg 4\pi\sigma$.
- Have found possible solution which is intermediate between extremes and thermodynamically consistent.

Remaining problems

Calculations do not fit experimental data:

Remaining problems

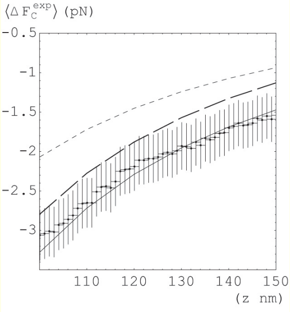
Calculations do not fit experimental data:



Experiment by Chen, Mohideen et.al. (Phys. Rev. B **76**, 035338)

Remaining problems

Calculations do not fit experimental data:



Experiment by Chen, Mohideen et.al. (Phys. Rev. B **76**, 035338)

Probable solution: free electrons don't behave as a plasma (?)

Remaining problems

Incomplete:

- Valid in high temperature/long separation domain. Theory to include electrodynamic screening would be nice.

Remaining problems

Incomplete:

- Valid in high temperature/long separation domain. Theory to include electrodynamic screening would be nice.
- Effects “leak” into $m > 0$ terms: must be manually prescribed away.

Thank you