JULIAN SCHWINGER AND THE CASIMIR EFFECT: THE REALITY OF ZERO-POINT ENERGY

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Julian Schwinger became interested in the Casimir effect in 1975. His original impetus was to understand the quantum force between parallel plates without the concept of zero point fluctuations of field quanta, in the language of source theory. He went on to consider applications to dielectrics and to spherical geometries in 1977. Although he published nothing on the subject in the following decade, he did devote considerable effort to understanding the connection between acceleration and temperature in the mid 1980s. During the last four years of his life, he became fascinated with sonoluminescence, and proposed that the dynamical Casimir effect could be responsible for the copious emission of photons by collapsing air bubbles in water.

1 Introduction

Julian Schwingerwas one of the pre-eminent theoretical physicists of the 20th Century. In addition to his famous solution of the problems of quantum electrodynamics, he made many other contributions to science. Nuclear physics, the theory of angular momentum, a reformulation of quantum kinematics, the quantum action principle, Euclidean field theory, many-particle Green's functions, the electro-weak synthesis, non-Abelian gauge theories, magnetic charge, source theory, and statistical models of atoms are some of the major themes of his work. (For a collection of some of his papers on these and other topics, see Ref. [1].) In this paper I document his contributions to the theory of the Casimir effect, an observable macroscopic consequence of quantum field theory, a subject on which Schwinger worked to the day of his death.

2 Casimir Effect

In 1975 Schwinger became interested in the Casimir effect through conversations with Seth Putterman.[2] (Conversations with Walter Dittrich may have also played a role.[3]) The Casimir effect is a fundamental aspect of quantum field theory, indeed of quantum mechanics, usually expressed as an observable consequence of the zero-point fluctuations of the normal modes of the electromagnetic field, or of whatever quantum fields are relevant. From another, complementary point of view, it is the macroscopic manifestation of the van

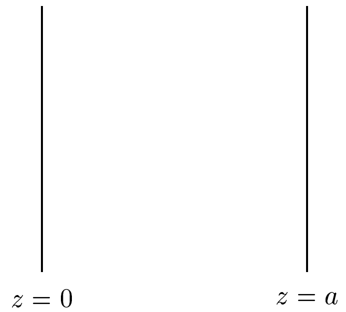

Figure 1: Parallel conducting plates separated by a distance a.

der Waals forces between the molecules that make up material bodies.[a] Let us begin by reviewing the history of this subtle yet fundamental phenomenon.

In 1948 H. B. G. Casimir [7] considered two perfectly conducting parallel plates in vacuum separated by a distance a. See Figure 1. Although it is usually, and correctly, asserted that the zero-point oscillations of the fields in the vacuum are unobservable, the presence of the boundaries changes that situation. On a perfect conductor, the tangential component of the electric field must be zero. Casimir observed that one could measure the *difference* between the zero-point energy in vacuum and the zero-point energy in the presence of the boundaries,

$$E_c = \sum \tfrac{1}{2}\hbar\omega_{\rm cond} - \sum \tfrac{1}{2}\hbar\omega_{\rm vac}, \tag{1}$$

where the subscripts refer to the normal modes, of frequency ω, in the presence of the conducting boundaries, and in the vacuum of unbounded space, respectively, and the sums are over all possible normal modes in the two situations. Of course, this formula is purely symbolic, since both sums are horribly divergent. In effect, Casimir gave a proper definition to this sum, and was able to extract a finite result, the Casimir energy for this geometry. Because the plates are assumed to extend indefinitely in the transverse directions (the x and y directions in the coordinate system shown in Fig. 1), it is the energy E_c

[a] Casimir and Polder in a heroic calculation derived the retarded van der Waals force using nonrelativistic quantum electrodynamics.[4] Shortly afterwards, Niels Bohr asked Casimir what he was doing. After hearing of this work, Bohr 'mumbled something about zero-point energy. He gave me a simple approach.' [5] That new approach was reported in Paris, with a rederivation of the force between molecules, and the force between a molecule and a conducting plane.[6] We will discuss these results below. The derivation of the force between conducting planes [7] followed shortly.

per unit area of the plates which Casimir calculated,

$$E_c = -\frac{\pi^2}{720}\frac{\hbar c}{a^3}, \tag{2}$$

or, by differentiating with respect to the plate separation a, the force per unit area f,

$$f = -\frac{\pi^2}{240}\frac{\hbar c}{a^4} = -0.013\frac{1}{a^4}\text{dyn } (\mu\text{m})^4/\text{cm}^2. \tag{3}$$

Note the minus sign in these expressions. That means that the Casimir force is attractive: The two plates are pulled toward each other. To this day, the sign of this effect defies intuitive understanding.

As early as 1956 experimental results appeared more or less in agreement with the Casimir theory.[8] It is very difficult to measure the Casimir force between conductors, because any small stray electrical charge on the conductors will give rise to a much larger electrical attraction or repulsion. (Very recently, however, definitive experimental results, completely consistent with Casimir's prediction, have been published.[9]) On the other hand, it was clear from the outset that there was nothing particularly special about having conducting boundaries. Starting in 1955, E. M. Lifshitz and collaborators in Moscow developed the theory of the Casimir effect for parallel dielectrics, that is for boundaries as shown in Fig. 1, but with different dielectric constants $\epsilon(z)$ in the three regions,

$$\epsilon(z) = \begin{cases} z < 0: & \epsilon_1 \\ 0 < z < a: & \epsilon_3 \\ a < z: & \epsilon_2 \end{cases}. \tag{4}$$

For this geometry Lifshitz et al. obtained a somewhat complicated formula[10] that expressed the force per unit area between the dielectrics in terms of the dielectric constants, which were assumed to be functions of the frequency:

$$f = -\frac{1}{8\pi^2}\int_0^\infty d\zeta \int_0^\infty dk^2 2\kappa_3 \left\{ \frac{1}{\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}e^{2\kappa_3 a} - 1} + \frac{1}{\frac{\kappa_3'+\kappa_1'}{\kappa_3'-\kappa_1'}\frac{\kappa_3'+\kappa_2'}{\kappa_3'-\kappa_2'}e^{2\kappa_3 a} - 1} \right\}, \tag{5}$$

where $\zeta = \frac{1}{i}\omega$ is the imaginary frequency, $k^2 = \mathbf{k}_\perp^2$ is the square of the transverse momentum, $\kappa^2 = k^2 + \zeta^2\epsilon$, and $\kappa' = \kappa/\epsilon$. In fact, this Lifshitz force was confirmed, based on knowledge of these dielectric constants, in a beautiful experiment of Sabisky and Anderson,[11] who measured the force holding a film of superfluid liquid helium to a SrF_2 substrate, to high precision, over distance scales differing by a factor of 1000. So by 1973 there was no doubt of the theoretical or experimental reality of the Casimir effect.[b]

As noted, in 1975, Schwinger became interested in explaining the Casimir effect in the source theory language, which 'makes no reference to quantum oscillators and their associated zero point energy.' [13] As usual, his presentation was first to his field theory class, and only then did he write a short publication.[13] Anticipating that the effect of the two polarizations of electromagnetism was merely a doubling of that for a single, massless, scalar mode, his derivation consisted, first, in obtaining the general expression for the infinitesimal change in the action under an infinitesimal change in the physical parameters,

$$\delta W = \frac{i}{2}\int (dx)(dx')D(x,x')\delta D^{-1}(x',x), \tag{6}$$

where D is the massless propagation function or Green's function, or the equivalent change in the energy

$$\delta\mathcal{E} = -\frac{i}{2}\int (d\mathbf{r})(d\mathbf{r}')d\tau D(\mathbf{r},\mathbf{r}',\tau)\delta D^{-1}(\mathbf{r}',\mathbf{r},-\tau), \tag{7}$$

which ignores transient effects. Then, by inserting an appropriate Green's function that satisfies the Dirichlet boundary conditions at $z=0,a$, written in terms of the longitudinal eigenfunctions, $\sqrt{2/a}\sin(n\pi z/a)$, he obtained the following formula for the change in the energy per unit area if the separation is changed by an amount δa, due to the Green's functions in the region $0<z<a$:

$$\frac{\delta\mathcal{E}_a}{A} = \frac{1}{4\pi}\frac{\delta a}{a}\frac{1}{i\tau}\frac{d^2}{d\tau^2}\frac{1}{1-e^{-i(\pi/a)\tau}}, \tag{8}$$

where the limit $\tau\to 0$ is understood. This result is divergent in that limit. But Schwinger then subtracted off the contribution from the region on the other side of the plate, $a<z<L$ (an additional conducting plate is placed at $z=L\gg a$), which may be immediately inferred from Eq. (8) to be

$$\frac{\delta\mathcal{E}_{L-a}}{A} = -\frac{1}{4\pi}\delta a\frac{1}{i\tau}\frac{d^2}{d\tau^2}\frac{1}{\pi i\tau}. \tag{9}$$

The force per unit area is then immediately found from the sum of Eq. (8) and Eq. (9) to be

$$f = -\frac{1}{A}\frac{\partial\mathcal{E}}{\partial a} = -\frac{\pi^2}{480}\frac{1}{a^4}, \tag{10}$$

[b]Schwinger noted in a talk in 1988 in honor of Herman Feshbach that Leonard Schiff had proposed that van der Waals forces were responsible for holding such a helium film to a surface, thus anticipating the Lifshitz theory.[12]

indeed, exactly one-half Casimir's result (3).

Schwinger concluded this note by rederiving the effect of finite temperature, in particular, the high-temperature limit,

$$kT \gg \frac{\pi}{a}: \quad f_T = -\frac{\zeta(3)}{8\pi}\frac{kT}{a^3}, \tag{11}$$

which had first been obtained by F. Sauer and J. Mehra.[14] Schwinger justified this publication, apart from it giving the Casimir effect a source theory context free from an operator substructure, by quoting from C. R. Hargreaves,[15] who stated that 'it may yet be desirable that the whole general theory be re-examined and perhaps set up anew.' The context of the latter remark was a discrepancy between the temperature dependence found between conducting plates, and that found from the temperature-dependent Lifshitz formula[10] when the dielectric constant in the region outside $0 < z < a$ is set equal to infinity, a process which should correspond to a perfect conductor. Unbeknownst to Schwinger, this error had been corrected subsequently.[16] (Hargreaves had corrected another error in Lifshitz' paper having to do with the effect of imperfect conductors.)

It was partly this (nonexistent) discrepancy, but primarily the challenge to understanding the phenomenon in his own language, that led Schwinger, and his postdocs Milton and DeRaad, to write 'Casimir Effect in Dielectrics,'[17] in which the Lifshitz formula for the Casimir force between parallel dielectrics was rederived in an elegant, action-principle based, Green's function technique. The key point here was that the effective product of electric fields could be represented in terms of the classical electromagnetic Green's dyadic,

$$\mathbf{E}(\mathbf{r})\mathbf{E}(\mathbf{r}')\bigg|_{\text{eff}} = \frac{\hbar}{i}\boldsymbol{\Gamma}(\mathbf{r},\mathbf{r}';\omega), \tag{12}$$

where the Green's dyadic satisfies

$$-\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{\Gamma}) + \omega^2\epsilon\boldsymbol{\Gamma} = -\omega^2\mathbf{1}\delta(\mathbf{r}-\mathbf{r}'). \tag{13}$$

From $\boldsymbol{\Gamma}$ Schwinger, Milton, and DeRaad calculated the change in the energy, using a method similar to that sketched above, or equivalently, the force directly from the electromagnetic stress tensor,

$$T_{zz} = \frac{1}{2}[H_\perp^2 - H_z^2 + \epsilon(E_\perp^2 - E_z^2)], \tag{14}$$

where $\mathbf{H}$ is calculated from $\mathbf{E}$ (and hence $\boldsymbol{\Gamma}$) using Maxwell's equations. Removing constant divergent terms from the result, the so-called volume stress, which would be present if a given dielectric extended over all space, they succeeded in rederiving the Lifshitz formula. As a special case, they took the perfect conductor limit noted above ($\epsilon \to \infty$ in the external region) and obtained the Casimir result, as well as the appropriate high and low temperature limits found by Sauer and Mehra.[14] They also showed how, in the case of tenuous dielectrics, i.e., in the case when $\epsilon - 1 \ll 1$, the Casimir force could be thought of as the superposition of the van der Waals attractions between the individual molecules (separated by a distance r) that made up the media,

$$\text{large separations:} \quad V = -\frac{23}{4\pi}\frac{\alpha_1\alpha_2}{r^7}, \tag{15}$$

$$\text{small separations:} \quad V = -\frac{3}{\pi r^6}\int_0^\infty d\zeta\alpha_1(\zeta)\alpha_2(\zeta), \tag{16}$$

where $\alpha = (\epsilon - 1)/4\pi N$ is the electric polarizability of the molecules, with number density N. These are the van der Waals potentials originally derived by Casimir and Polder[4] and by Fritz London,[18] respectively.

These results were all explicitly contained in the much earlier papers by Lifshitz and collaborators,[10] to whom due acknowledgement was made. Nevertheless, Lifshitz was somewhat offended by this paper, and he wrote Schwinger a letter: 'Thank you for the preprint of your ... paper .... It was gratifying to know of your interest in my earlier work.'

'Of course, the method adopted in this paper is far superior than [sic] the method which was used in my first paper of 1954. But it seems to me that it is almost identical with the method developed later by I. Dzyaloshinskii, L. P. Pitajevskii, and myself. The derivation of my results by this method was published in our joint paper in *Advances of Physics*, 1961 (identical with the paper in *Soviet Physics, Uspechi*, referred to in your preprint); it was also reproduced in the book by Abrikosov, Gorkov, Dzyaloshinskii on the *Field Theoretical Methods in Statistical Physics* (English translation, Prentice Hall, 1963).'

'As to the formula for the low temperature limit of the force between the two perfect metallic surfaces (formula 3.17 of your paper), the error in sign in my paper was the result merely of an unfortunate slip in rewriting the Euler summation formula, and not of a deeper origin. This error has since [been] noticed by different authors both in our country and elsewhere.' [16]

The only really new result in this paper was an attempt to derive the surface tension for an ideal liquid (liquid helium) from such considerations, by examining the effect of a change of shape of the surface on the energy. 'The second-order change in the energy ... is directly related to the surface tension.' [17] Unfortunately, a quadratically divergent result was obtained. However, with reasonable numbers inserted to provide a physical cutoff to the divergence, a value for the surface tension, and for the latent heat, could be obtained crudely in agreement with the observed values to within a factor of two or three. This idea remains provocative yet unresolved.

A few months later the same three authors wrote a second paper on Casimir phenomena, entitled 'Casimir Self-Stress on a Perfectly Conducting Spherical Shell.' [19] The impetus for this work went back to another paper of Casimir, this one in 1956, in which he suggested that the attractive Casimir force could balance the Coulomb repulsion of a semiclassical model of an electron.[20] More precisely, it had long been known that a purely electromagnetic classical model of an electron was impossible, that one had to add the so-called Poincaré stresses to stabilize the particle. Casimir now suggested that those stresses could arise from quantum mechanics. Indeed, if a reasonable guess extrapolated from the parallel plate calculation was used, one could calculate a value for the charge on the electron, or better, the fine structure constant, $\alpha = e^2/\hbar c$, consistent, perhaps, with the experimental value, $\alpha = (137.036\ldots)^{-1}$.

It remained for Timothy Boyer, a student of Sheldon Glashow at Harvard, to take up the challenge of a real calculation for the spherical geometry in 1965. He calculated the change in the zero-point energy due to the presence of a perfectly conducting spherical shell of radius a. Both modes interior to and exterior to the shell had to be included in order to get a finite result. This impressive calculation was difficult and subtle, and involved extensive numerical calculation. His result, obtained after three years of work, was accurate to only one significant figure, but it was of the *opposite* sign compared to the one found by Casimir in the parallel geometry: [21]

$$E_B = +\frac{0.9}{2a}. \tag{17}$$

His expression was subsequently evaluated more accurately, to three significant figures, by Davies.[22]

Because this result was so surprising, and devastating to Casimir's electron model, it was an obvious target for a recalculation by Schwinger and his postdocs, now that their improved Green's function machinery had been honed. By the end of 1977 they had derived a compact formula for the Casimir energy of a conducting shell, much simpler than that of Boyer,

$$E = -\frac{1}{2\pi a}\sum_{l=1}^{\infty}(2l+1)\frac{1}{2}\int_{-\infty}^{\infty} dy\, e^{i\epsilon y} x \frac{d}{dx}\log(1-\lambda_l^2), \tag{18}$$

where the sum is taken over the different angular momentum modes, the integral is over (imaginary) frequencies, $y = \frac{1}{i}\omega a$, the quantity $x = |y|$, and the logarithm depends on

$$\lambda_l(x) = (s_l e_l)'(x) \tag{19}$$

(where the prime denotes differentiation). The functions e_l and s_l are given in terms of modified Bessel functions,

$$s_l(x) = \sqrt{\frac{\pi x}{2}} I_{l+1/2}(x), \tag{20}$$

$$e_l(x) = \sqrt{\frac{2x}{\pi}} K_{l+1/2}(x). \tag{21}$$

The expression (18), which is formally divergent, has been regulated by evaluating the underlying Green's function at unequal times, $t = t' + \tau$, i.e., by 'time-splitting.' At the end of the calculation one is to take the limit $\epsilon = \tau/a \to 0$. Unfortunately, at this point Milton and DeRaad had a bit of difficulty in seeing how to extract a number from this formula, so a few months passed. (Schwinger had contented himself with deriving the formula.) Unfortunate, because just at that point a paper by Balian and Duplantier appeared,[23] who obtained a different formula, based on a multiple scattering formalism, and obtained a result, consistent with Boyer's number, but now accurate to three significant figures. So the postdocs worked hard, discovered how to extract a reliable answer based on the use of uniform asymptotic approximations (the first term of which was accurate to 2%, while Balian and Duplantier's first approximation was only accurate to 8%), and obtained the result accurate to *five* significant figures,

$$E = \frac{0.92353\underline{1}}{2a}. \tag{22}$$

The reaction from Boyer and Balian was rather unexpected. In a letter to Lester DeRaad (DeRaad and Boyer, of course, had been fellow graduate students at Harvard) Tim Boyer wrote, 'The calculations presented seem sophisticated, and presumably are carefully done. However, the comments on my work in the text of the Casimir sphere paper are hardly generous; my colleagues would characterize them differently.' [24]

He went on to apprise DeRaad of the Davies calculation, and to give further experimental references, which were incorporated into the published papers. In addition, an appreciative comment about Boyer's work was inserted into Ref. [19].

Roger Balian wrote Schwinger to say 'I guess it would be interesting to compare our respective approaches, which have the common feature of being based on the elimination of fields and consideration of sources. Our formalism was mainly intended to deal with arbitrary geometries; it is based on an expansion which converges rapidly in cases of interest (slightly deformed conducting sheet, spherical shell, etc.. . .). However, we construct the electric Green's function in terms of fictitious monopole currents, and restrict to conductors. Your approach has the advantage of allowing the treatment of dielectrics; I do not see, however, how to use it for arbitrary geometries; on the other hand, would you obtain instabilities of the surface of a dielectric at $T \neq 0$, thus generalizing the effect which we pointed out for a conducting foil?' [25] Since this letter was dated December 28, 1977, more than five months before Schwinger's paper on the Casimir effect for a sphere was submitted,[19] it seems likely that at that point Balian had only seen the dielectric paper; [17] hence the remark about geometries.

Milton responded to both of these letters graciously, and promised to look at Balian's technical points in the future, but that never occurred.

Schwinger's papers on the Casimir effect were influential, not for their explicit results, which, as we have seen, were mostly well-known, but for the development of powerful techniques of attacking such problems, which continue to be exploited. A recent example is the study of the dimensional dependence of the Casimir effect in hyperspheres by Bender and Milton.[26]

3 Acceleration and Temperature

In the mid 1980s Schwinger received an honor, the Monie Ferst Medal given by the Georgia Institute of Technology chapter of Sigma Xi. The associated Symposium, held on May 20, 1986, consisted of technical talks by three of his former students, Milton, Ken Johnson, and Margaret Kivelson, and a provocative talk by Schwinger on 'Accelerated Observers and the Thermal Power Spectrum of the Vacuum.' This talk reported his work on the 'Unruh' effect.[27][c] Although he had spent considerable time working on this project, and presented a most interesting presentation on the subject, he did not then, or later, ever write up this work. All that was printed is his abstract for the symposium: 'Source theory, with its foundation in idealizations of particle emitters and absorbers (detectors), provides a natural, self-contained approach that is intermediate between the mathematical attitudes of quantum field theorists and the physical consideration of specific detection mechanisms. The periodicity inherent in the circular coordinate form of the Euclidean Green's function, as transformed into hyperbolic (Rindler) coordinates, immediately yields the characteristic property of a thermal Green's function. The explicitness with which this can

[c]This interest may have been sparked by a 1983 letter from Kirk McDonald of Princeton, on the Unruh effect.[12]

be done assists in recognizing that, despite the thermal nature of the spectrum, there are definite phase relations that would show up in other experiments.' [28]

The Schwinger Collection at UCLA does possess a few manuscripts on the subject of acceleration and radiation which Schwinger started but did not complete.[12] There is also a draft of a paper with Manuel Villasante, dated 1994, entitled 'Acceleration, Black Holes, and Temperature;' this paper was never submitted, presumably because of Schwinger's illness.[d] Villasante received his Ph.D. under Robert Finkelstein's direction, and became Schwinger's final postdoc. However, they never completed a paper together. The first part of the extant 56 page manuscript is essentially equivalent to the earlier solo effort of Schwinger on accelerated detectors; Villasante's contribution largely consisted of extending the ideas to a Schwarzchild space, hoping thereby to make the connection with Hawking radiation.[29] In fact, Villasante recalls they only had one brief conversation about the work, late at night, and Schwinger never responded to his messages.[30][e] Schwinger apparently never felt this work was complete enough.[33]

4 Casimir effect and sonoluminescence

Schwinger's last physics endeavor marked a return to the Casimir effect, of which he had been enamored nearly two decades earlier. It was sparked by the remarkable discovery of single-bubble sonoluminescence. It was not coincidental that the leading laboratory investigating this phenomenon was, and is, at UCLA, led by erstwhile theorist Seth Putterman, long a friend and confidant. Putterman and Schwinger shared many interests in common, including appreciation of fine wines, and they shared a similar iconoclastic view of the decline of physics. So, of course, Schwinger heard about this remarkable phenomenon from the horse's mouth, and was greatly intrigued.

What is sonoluminescence? The word means the conversion of sound into light. As such, it had been observed since the thirties,[34] but this so-called multiple bubble sonoluminescence was hardly investigated, and was nearly completely forgotten by the last decade of this century. Not completely, because Tom Erber, on one of his many visits to UCLA, told Putterman about this old effect,[2] and, in short order, a much more remarkable version of the effect was discovered. If a single bubble of air is injected into a beaker of water, and

[d] Schwinger died of pancreatic cancer in July 1994, having been diagnosed in February of that year.

[e] Schwinger also had a Greek student in his last years, Evangelos Karagiannis, who did his Ph.D on a related topic,[31] but 'he also found it impossible to get hold of Schwinger for anything.'[30] What these late collaborators failed to appreciate was that 'Schwinger was hard to work with if you wanted guidance, but easy to work with if you wanted inspiration.'[32]

held in a node of a standing acoustic wave set up in the water, the bubble will begin to expand and contract in concert with the frequency of the standing wave. If the ultrasonic wave has a frequency of about 20,000 Hertz, and a pressure amplitude of about one atmosphere, a small suspended bubble of air will expand and collapse 20,000 times a second, undergoing a change in radius of a factor of 10 or more (and hence, in volume, of at least a factor of 1,000), from, say 4×10^{-3} cm to 4×10^{-4} cm. If the parameters are chosen just right (including a small percentage of noble gas, for example, the amount of argon in our atmosphere, seems essential), exactly at minimum radius a bright flash of light is released from the bubble. This flash of light consists of approximately one million optical photons, so that about 10 MeV of energy is converted into light on each collapse. This flash of light, integrated over many cycles, is bright enough to be visible to the naked eye if the water is observed in a darkened room. (The author has seen this effect for himself.) Whatever produces the flash of light is sufficiently non-catastrophic that it does not in any way disrupt the bubble, and the periodic collapse and re-expansion continues for many minutes, perhaps months. For a review of the experimental situation, see Ref. [35].

The hydrodynamics of the bubble collapse and re-expansion appears to be quite well understood. What is not understood at all is how some of the energy in the bubble, extracted from the sound field, is converted into the intense flash of light. The duration of the flash has not been determined, but it is less that 10^{-11} seconds, much smaller than the period of the bubble collapse, but apparently long compared to the period of optical photons (about 10^{-15} seconds). Although there have been various classical and quantum hypotheses put forward, they tend not to be, in the words of Putterman, 'falsifiable.'[35]

Of course, Putterman told Schwinger about the phenomenon right away. He called Schwinger at home, and immediately Schwinger drove down to see it. At first Schwinger had difficulty in seeing the faintly glowing bubble. Putterman told him to 'look at $r = 0$,' and soon he saw the bubble at the center of the spherical vessel. Schwinger's reaction was 'I'm shaken.' He at once started work on the problem of understanding what was happening.[2]

Schwinger immediately had the idea that a dynamical version of the Casimir effect might play a key role. In a letter to Putterman 'Re: nanosecond sonoluminescence' wherein he proposes the Casimir effect mechanism, Schwinger opens with a quotation, presumably written on Martin Luther King Day: 'MLK: "I have a dream." JS: "I have a feeling."'[12] The idea was that the virtual photons present, due to the Casimir effect, or in conventional language, vacuum fluctuations, in a bubble in a dielectric medium could be converted into real photons because the radius of the bubble is rapidly changing. This is, in fact, presumably closely related to the Unruh effect [27] in which a moving mirror radiates a black body spectrum of photons—in turn closely allied with Hawking radiation from a black hole.[29] (Recall that Schwinger had worked for a while on the Unruh effect in the mid 1980s into the 1990s, although he never completed a paper on the subject.[f]) So there were two challenges for Schwinger. One was to develop the 'dynamical Casimir effect' for the spherical geometry of a bubble, and the second was to apply that effect to the hydrodynamic situation of a collapsing bubble in sonoluminescence.

The first step was initiated[g] through two papers Schwinger published in *Letters in Mathematical Physics*, edited by the frequent UCLA visitor Moshé Flato, as sequels to his first, 1975, Casimir paper,[13] also published in the same journal. In the first,[37] he derived the original Casimir effect for parallel conducting plates by an elegant proper-time approach, while in the second,[38] he reconsidered dielectric slabs. In both cases, the emphasis was on energy rather than force. He followed[h] this by two somewhat longer articles in the *Proceedings of the U. S. National Academy of Sciences.* (Unlike Feynman,[39] Schwinger continued throughout his career to find the Academy a useful scientific venue.) In the first, he rederived, for the third time, the Lifshitz theory for the Casimir effect between parallel dielectric slabs,[40] in an efficient way making use of an explicit break-up into Transverse Electric (TE) and Transverse Magnetic (TM) modes. As had been done in his earlier collaborative work,[17] he explicitly removed volume and surface energies: 'One finds contributions to E [the energy] that, for example, are proportional ... to the volume enclosed between the slabs. The implied constant energy density—independent of the separation of the slabs—violates the normalization of the vacuum energy density to zero. Accordingly, the additive constant has a piece that maintains the vacuum energy normalization. There is also a contribution to E that is proportional to [the area], energy associated with individual slabs. The normalization to zero of the energy of an isolated slab is maintained by another part of the additive constant.' [40] Then he turned to the case of interest for sonoluminescence, spherical dielectrics. In 'Casimir Energy for Dielectrics: Spherical Geometry' [41]

[f] In 1990, just before single bubble sonoluminescence was discovered, Schwinger wrote a manuscript entitled 'Superluminal Light' (a later version was called 'Tachyonic Light') which was a reaction to the claim by K. Scharnhorst and G. Barton that light speeds greater than the speed of light in vacuum are possible in a parallel plate capacitor, the original Casimir effect geometry, indeed as an induced consequence of the Casimir effect.[36] Unlike them, Schwinger found the effect was nonuniform, dispersive (that is, frequency dependent), and that the effect persisted if only a single plate was present.[12]

[g] Early in the process he gave the talk 'A Progress Report: Energy Transfer in Cold Fusion and Sonoluminescence.' The title here hearkens back to Oppenheimer's seminar given when Schwinger first came to Berkeley in 1939.[12]

[h] Actually Ref. [40] was submitted essentially simultaneously with Ref. [38].

he began an elegant treatment of the Casimir effect in that situation. Unfortunately, he only treated the TE modes, and went only far enough to see that the parallel geometry result is recovered if a careful limit of the radius of the sphere going to infinity is taken. Explicitly, he left the details to Harold![i]

But Harold, or Sagredo,[42] had been over this ground already. Thirteen years earlier, while still at UCLA, Milton had computed the Casimir effect for a dielectric ball.[43] Perhaps Schwinger can be forgiven his ignorance of his former student and postdoc's work by the fact that this paper was completed and published after Milton had gone to Ohio State.[j] In any case, Schwinger did not get far enough with this calculation to apply it to sonoluminescence. Instead, when he started to develop his theory of sonoluminescence in a series of five papers[k] in the *Proceedings of the U. S. National Academy of Sciences*[44,45] he simply wrote down a naive approximation for the Casimir energy obtained, in effect, by subtracting the zero-point energy of the vacuum from that for the medium, giving the quartically divergent formula,

$$E_{\text{bulk}} = \frac{4\pi a^3}{3}\int\frac{(d\mathbf{k})}{(2\pi)^3}\frac{1}{2}k\left(1-\frac{1}{n}\right), \tag{23}$$

where n is the index of refraction of the medium. Schwinger had forgotten his own injunction of subtracting off the volume energy, that term which 'would be present if either medium filled all space.' Since this expression is very divergent, it is extraordinarily sensitive to the cutoff which must be used on physical grounds to give a finite result. However, if a plausible ultraviolet cutoff is used, Schwinger obtained a sufficiently large Casimir energy, $E_{\text{bulk}} \sim 10$ MeV.

The problem is that the bulk energy Schwinger considered is not relevant to sonoluminescence.[43] It is, in fact, a kind of self energy, one that contributes to the density of the water, and of the gas, that is already phenomenologically described. As further noted above, the same is true of the surface energy, it being subsumed into the definition of the surface tension. The correct conclusion from the calculation of Refs. [43] and [46] is that the Casimir energy is very small,

$$E_c = \frac{23(n-1)^2}{384a}, \tag{24}$$

[i] Harold, introduced as an acronym for the 'hypothetical alert reader of limitless dedication' in Ref. [42], was the name of Schwinger's older brother.

[j] However, in Milton's last meeting with Schwinger, in December 1993, Schwinger did not wish to be reminded of this earlier work.

[k] There are notes for at least three further papers in Schwinger's files on 'Casimir Light,' the last being subtitled 'A Study in Green.' These must represent his last scientific work.[12]

which only amounts to about 10^{-3} eV in the case of sonoluminescing bubbles (as well as having the wrong sign), and therefore is completely irrelevant to sonoluminescence.[l]

Schwinger's final paper, on sonoluminescence,[45] was published in the month of his death. As we noted he was typically unaware of some of his colleagues' own papers relevant to the subject, but, atypically, he was very explicitly seeking Milton's collaboration in the last year of his life (Milton talked to him at some length in December 1993, at the annual Christmas party given by the Alfredo Baños', which he and Clarice often attended,[m] and at a subsequent lunch). He also arranged to have the earlier papers, on Casimir energy, but not the later ones, on sonoluminescence, sent to Milton. Schwinger felt that 'carrying out that program is—as one television advertiser puts it—job one.'[48] It seems apparent that he was aware of the inadequacies of his treatment of the Casimir effect, and was looking for additional expertise and strength. The subject is not completely closed, because there are serious subtleties in these Casimir calculations, the adiabatic approximation (that is, treating the bubble radius as slowly varying on an electromagnetic time scale) may be invalid, and most likely a shock forms, which allows for discontinuities on very short time scales. So Schwinger's ideas here are still being explored.[49] Perhaps Julian Schwinger will ultimately have the last laugh!

5 Conclusions

As we have noted, Schwinger explicitly and implicitly drew parallels between cold fusion and sonoluminescence. At first blush this seems implausible. After all, sonoluminescence without doubt exists, while cold fusion does not. But Schwinger's point was one of overcoming seemingly impossibly different scales. In the case of cold fusion, how can the Coulomb barrier be overcome at very low energies; in the case of sonoluminescence, how could hydrodynamics, characterized by acoustic phonons, couple to quantum electrodynamics, characterized by much higher energy photons? It is natural that he would find the attempt to solve these conundrums challenging. And, as it became increasingly untenable to pursue cold fusion, he shifted his efforts toward the experimentally confirmed sonoluminescence.

Seth Putterman recounts his final meeting with Schwinger two days before his death. Schwinger did not want to talk about history, but about physics, and wanted to know what was new in sonoluminescence. Putterman told him

[l] Schwinger did treat briefly the production of photons through the instantaneous collapse of the bubble. This will be discussed in another contribution to this Proceedings.[47]
[m] His role was to hide the three kings in the Christmas tree.

of the puzzling fact that water is the 'friendliest' liquid for the phenomenon, and that the effect only appears if about 1% noble gas is present. Schwinger thought for a bit, and said, 'It probably has something to do with evolution.'[2] Heady stuff indeed!

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