Thermal corrections to the Casimir effect

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Abstract. The Casimir effect, reflecting quantum vacuum fluctuations in the electromagnetic field in a region with material boundaries, has been studied both theoretically and experimentally since 1948. The forces between dielectric and metallic surfaces both plane and curved have been measured at the 10 to 1 percent level in a variety of room-temperature experiments, and remarkable agreement with the zero-temperature theory has been achieved. In fitting the data various corrections due to surface roughness, patch potentials, curvature, and temperature have been incorporated. It is the latter that is the subject of the present article. We point out that, in fact, no temperature dependence has yet been detected, and that the experimental situation is still too fluid to permit conclusions about thermal corrections to the Casimir effect. Theoretically, there are subtle issues concerning thermodynamics and electrodynamics which have resulted in disparate predictions concerning the nature of these corrections. However, a general consensus has seemed to emerge that suggests that the temperature correction to the Casimir effect is relatively large, and should be observable in future experiments involving surfaces separated at the few micrometer scale.

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1. Introduction

About the same time that Schwinger and Feynman were inventing renormalized quantum electrodynamics, Casimir discovered that quantum electrodynamic fluctuations resulted in macroscopic forces between conductors and dielectrics [1]. The theory was a natural outgrowth of the Casimir-Polder theory of the retarded dispersion force between molecules [2]. The general theory for the forces between parallel dielectrics was worked out by Lifshitz and collaborators [3], who also included temperature corrections, which were considered further by Sauer [4] and Mehra [5]. Some years later, the whole theory was rederived by Schwinger, DeRaad, and Milton [6].

The early experiments on Casimir forces were rather inconclusive – for a review see [7]. However, the corresponding Lifshitz theory was verified rather impressively by Sabisky and Anderson [8], so there could hardly be any doubt of the validity of the essential ideas. Starting about a decade ago, modern experiments by Lamoreaux [9, 10, 11, 12], Mohideen and collaborators [13, 14, 15], and by Erdeth [16] brought the experimental measurement of the Casimir force between curved metal surfaces (mapped to the plane geometry by the proximity approximation [17, 18]) into the percent accuracy region. (Exact results have now apparently rendered the use of the proximity approximation, which cannot be extended beyond leading order, unnecessary. See, for example, [19, 20, 21].) Application of such Casimir forces to nanoelectromechanical devices have been suggested by experiments at Bell Labs and Harvard [22, 23, 24]. Only one experiment so far, of limited accuracy ($\sim 15\%$), has employed parallel plates [25]. The difficulty of maintaining parallelism in that geometry limits the accuracy of the experiment, but the forces are much larger than those between a sphere and a plate, so the forces can, in principle, be determined at much larger separations. Proposals to perform measurements of the force between a cylinder and a plane [26] and between eccentric cylinders [27] have advantages because the forces are stronger than between a sphere and a plane, yet the difficulties in assuring parallelism are not so severe as with two plane surfaces. The most precise experiments so far, based on both static and dynamical procedures between a plate and a spherical surface, have been performed at Purdue [28, 29, 30], where the accuracy is claimed to be better than 1% at separations down to less than 100 nm.

All present experiments agree well with the zero-temperature Casimir theory when surface roughness and finite conductivity corrections are included [31, 32]. The issue about which controversy has recently erupted is the temperature dependence. (For recent statements of both sides of the controversy, see [30, 33, 34, 35, 36].) All experiments reported to date have been conducted at room temperature, so there is no direct evidence for or against any particular model of the temperature dependence. Indirect evidence for this dependence has been inferred based on the nonzero shift in the theoretical Casimir force between the surfaces due to the difference between the force at zero temperature and at 300 K. Surprisingly, this temperature shift is not so straightforwardly computed as one would have at first suspected.

It is the purpose of the present paper to frame the question of the temperature dependence of the Casimir force in the context of the history of the subject and the present experimental constraints, as well as to point out ways of reconciling the ambiguities both from the theoretical and experimental sides. In the following section we review the standard approach given in [6] for both dielectric and metal surfaces. Then, in section 3 we give the arguments why the transverse electric (TE) zero mode should not be included, and how this impacts the temperature dependence of the force, and the resulting impact on the free energy and entropy. Other theoretical arguments for and against this point of view are discussed in section 4. The status of the experimental situation, and the possibility of dedicated experiments to search for the temperature dependence of the Casimir effect, will be reviewed in section 5. Finally, some new calculations are presented in section 6 in the hope of providing signatures to help resolve the controversy.

2. Conventional temperature approach

The zero-temperature Casimir effect between parallel conducting plates, or between parallel dielectrics, is very well understood, and is not controversial. The formula for the latter, which includes the former as a singular limit, may be derived by a multitude of formalisms, which will not be reviewed here [6, 31, 37, 38, 39]. For a system of parallel dielectric media, characterized by a permittivity

$$\varepsilon(z) = \begin{cases} \varepsilon_1, & z < 0, \\ \varepsilon_3, & 0 < z < a, \\ \varepsilon_2, & a < z, \end{cases} \tag{2.1}$$

where the various permittivities are functions of frequency, the Lifshitz force per unit area on one of the surfaces is at zero temperature

$$P^{T=0} = -\frac{1}{4\pi^2}\int_0^\infty \mathrm{d}\zeta \int_0^\infty \mathrm{d}k_\perp^2 \kappa_3 (d^{-1} + d'^{-1}), \tag{2.2}$$

where ζ is the imaginary frequency, $\zeta = -\mathrm{i}\omega$, and the longitudinal wavenumber is

$$\kappa_i = \sqrt{k_\perp^2 + \zeta^2 \varepsilon_i(\mathrm{i}\zeta)}, \tag{2.3}$$

while the transverse electric (TE) and transverse magnetic (TM) Green's functions are characterized by the denominators

$$d = \frac{\kappa_3 + \kappa_1}{\kappa_3 - \kappa_1}\frac{\kappa_3 + \kappa_2}{\kappa_3 - \kappa_2}\mathrm{e}^{2\kappa_3 a} - 1, \qquad d' = \frac{\kappa_3' + \kappa_1'}{\kappa_3' - \kappa_1'}\frac{\kappa_3' + \kappa_2'}{\kappa_3' - \kappa_2'}\mathrm{e}^{2\kappa_3 a} - 1, \tag{2.4}$$

respectively, where $\kappa_i' = \kappa_i/\varepsilon_i$.

The attractive Casimir pressure between parallel perfectly conducting planes separated by a vacuum space of thickness a is obtained by setting $\varepsilon_{1,2} \to \infty$ and $\epsilon_3 = 1$. In that case the TE and TM contributions are equal, and we have

$$P_C = -\frac{1}{8\pi^2}\int_0^\infty \mathrm{d}\zeta \int_{\zeta^2}^\infty \mathrm{d}\kappa^2 \frac{4\kappa}{\mathrm{e}^{2\kappa a} - 1} = -\frac{1}{\pi^2}\int_0^\infty \mathrm{d}\zeta \frac{\zeta^3}{\mathrm{e}^{2\zeta a} - 1} = -\frac{\pi^2 \hbar c}{240 a^4}, \tag{2.5}$$

which is Casimir's celebrated result [1].

The controversy surrounds the question of how to incorporate thermal corrections into the latter result. At first glance, the procedure to do this seems straightforward. It is well-known that thermal Green's functions must be periodic in imaginary time, with period $\beta = 1/T$ [40]. This implies a Fourier series decomposition, rather than a Fourier transform, where in place of the imaginary frequency integral in (2.5) we have a sum over Matsubara frequencies

$$\zeta_m^2 = \frac{4\pi^2 m^2}{\beta^2}, \tag{2.6}$$

that is, the replacement

$$\int_0^\infty \frac{\mathrm{d}\zeta}{2\pi} \to \frac{1}{\beta}\sum_{m=0}^{\infty}{}', \tag{2.7}$$

the prime being an instruction to count the $m = 0$ term in the sum with half weight. This prescription leads to the following formula for the Casimir pressure between perfect conductors at temperature T,

$$P^T = -\frac{1}{4\pi\beta a^3}\sum_{m=0}^{\infty}{}' \int_{nt}^\infty y^2\,\mathrm{d}y \frac{1}{\mathrm{e}^y - 1}, \tag{2.8}$$

where

$$t = \frac{4\pi a}{\beta}. \tag{2.9}$$

From this it is straightforward to find the high and low temperature limits,

$$P^T \sim -\frac{1}{4\pi\beta a^3}\zeta(3) - \frac{1}{2\pi\beta a^3}\left(1 + t + \frac{t^2}{2}\right)\mathrm{e}^{-t}, \qquad \beta \ll 4\pi a, \tag{2.10a}$$

$$P^T \sim -\frac{\pi^2}{240a^4}\left[1 + \frac{16}{3}\frac{a^4}{\beta^4} - \frac{240}{\pi}\frac{a}{\beta}\mathrm{e}^{-\pi\beta/a}\right], \qquad \beta \gg 4\pi a. \tag{2.10b}$$

These are the results found by Sauer [4] and Mehra [5], and by Lifshitz [3]. The two limits are connected by the duality symmetry found by Brown and Maclay [41]. The pressure may be obtained by differentiating the free energy,

$$P = -\frac{\partial}{\partial a}F, \tag{2.11}$$

which takes the following form for low temperature (now omiting the exponentially small terms)

$$F \sim -\frac{\pi^2}{720a^3} - \frac{\zeta(3)}{2\pi}T^3 + \frac{\pi^2}{45}T^4 a, \qquad aT \ll 1, \tag{2.12}$$

from which the entropy follows,

$$S \sim -\frac{\partial}{\partial T}F \sim \frac{3\zeta(3)}{2\pi}T^2 - \frac{4\pi^2}{45}T^3 a, \qquad aT \ll 1, \tag{2.13}$$

which vanishes as T goes to zero, in accordance with the third law of thermodynamics, the Nernst heat theorem.

3. Exclusion of TE zero mode

However, there is something peculiar about the procedure adopted above for a perfect metal. (This seems first to have been appreciated by Boström and Sernelius [42].) It has to do with the transverse electric mode of zero frequency, which we shall refer to as the TE zero mode. If we examine the zero frequency behavior of the reflection coefficients for a dielectric appearing in (2.4), we see that providing $\zeta^2\varepsilon(\mathrm{i}\zeta) \to 0$ as $\zeta \to 0$, the longitudinal wavenumber $\kappa_i \to k$ as $\zeta \to 0$, and hence $d \to \infty$ as $\zeta \to 0$. This means that there is no TE zero mode for a dielectric. This statement does not appear to be controversial [43]. However, if a metal is modeled as the $\varepsilon \to \infty$ limit of a dielectric, the same conclusion would apply. Because that would spoil the concordance with the third law noted in the previous section, the prescription was promulgated in [6] that the $\varepsilon \to \infty$ limit be taken before the $\zeta \to 0$ limit. But, of course, a real metal is not described by such a mathematical limit, so we must examine the physics carefully.

A simple model for the dielectric function is the plasma dispersion relation,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \tag{3.1}$$

where ω_p is the plasma frequency. For this dispersion relation, the condition $\zeta^2\varepsilon(\mathrm{i}\zeta) \to 0$ fails to hold as $\zeta \to 0$, and the idealized prescription result, namely the contribution of the TE zero mode, holds. However, real metals are not well described by this dispersion relation. Rather, the Drude model,

$$\varepsilon(\mathrm{i}\zeta) = 1 + \frac{\omega_p^2}{\zeta(\zeta+\gamma)}, \tag{3.2}$$

where the relaxation frequency γ represents dispersion, very accurately fits optical experimental data for the permittivity for $\zeta < 2\times 10^{15}$ rad/s [44, 45]. For example, for gold, appropriate values of the parameters are [46]

$$\omega_p = 9.03 \text{ eV}, \qquad \gamma = 0.0345 \text{ eV}. \tag{3.3}$$

In this case, the arguments given above for the exclusion of the TE zero mode apply.

The arguments are a bit subtle [38, 39], so we review and extend them here. Let us write the Lifshitz formula at finite temperature in the form

$$P^T = \sum_{m=0}^{\infty}{}' f_m = \int_0^\infty \mathrm{d}m\, f(m) - \sum_{k=0}^{\infty} \frac{B_{2k}}{(2k)!} f^{(2k-1)}(0), \tag{3.4}$$

where the second equality uses the Euler-Maclaurin sum formula, in terms of

$$f(m) = -\frac{1}{2\pi\beta}\int_0^\infty \mathrm{d}k_\perp^2\, \kappa(\zeta_m)\left(d_m^{-1} + d_m'^{-1}\right), \tag{3.5}$$

according to (2.2) and (2.7), where we assume that vacuum separates the two plates so $\kappa_3(\zeta_m) = \kappa(\zeta_m) = \sqrt{k_\perp^2 + \zeta_m^2}$. Here the denominators (2.4) are functions of ζ_m. By changing the integration variable from m to ζ_m we immediately see that the integral term in the Euler-Maclaurin sum formula corresponds precisely to the zero-temperature result (2.2).

One must, however, be careful in computing the low temperature corrections to this. One cannot directly expand the denominator d in powers of ζ because the $k_\perp$ integral in (3.5) ranges down to zero. Let us rewrite the TE term there as follows:

$$f^{(\mathrm{TE})}(m) = -\frac{1}{\pi\beta}\int_{2m\pi/\beta}^\infty \mathrm{d}\kappa\,\kappa^2 \left\{ \left[\frac{1+\sqrt{1+\zeta_m^2(\varepsilon(\mathrm{i}\zeta_m)-1)/\kappa^2}}{1-\sqrt{1+\zeta_m^2(\varepsilon(\mathrm{i}\zeta_m)-1)/\kappa^2}}\right]^2 \mathrm{e}^{2\kappa a}-1\right\}^{-1}. \tag{3.6}$$

Evidently, for the Drude model, or more generally, whenever

$$\lim_{\zeta\to 0}\zeta^2[\varepsilon(\mathrm{i}\zeta)-1]=0, \tag{3.7}$$

$f^{(\mathrm{TE})}(0)=0$. However, it is important to appreciate the physical discontinuity between $m=0$ and $m=1$ for room temperature. At 300 K, while $\zeta_0=0$, $\zeta_1=2\pi T=0.16$ eV, large compared the relaxation frequency γ. Therefore, for $m>0$,

$$\begin{aligned} f^{(\mathrm{TE})}(m) &\approx -\frac{1}{\pi\beta}\int_{\zeta_m}^\infty \mathrm{d}\kappa\,\kappa^2\left[\left(\frac{\sqrt{1+\omega_p^2/\kappa^2}+1}{\sqrt{1+\omega_p^2/\kappa^2}-1}\right)^2\mathrm{e}^{2\kappa a}-1\right]^{-1} \\ &\approx -\frac{1}{\pi\beta}\int_{\zeta_m}^\infty \mathrm{d}\kappa\,\kappa^2\frac{1}{\mathrm{e}^{2\kappa a}-1}, \end{aligned} \tag{3.8}$$

provided the significant values of ζ_m and κ are small compared the the plasma frequency ω_p. This is just the ideal metal result contained in (2.8). Insofar as this is accurate, this expression yields the low- and high-temperature corrections seen in (2.10b), (2.10a). However, there is now a discontinuity in the function $f^{(\mathrm{TE})}$. As $\zeta_m\to 0$,

$$f^{(\mathrm{TE})}(m)\to -\frac{1}{\pi\beta}\int_0^\infty \mathrm{d}\kappa\frac{\kappa^2}{\mathrm{e}^{2\kappa a}-1} = -\frac{\zeta(3)}{4\pi\beta a^3}, \tag{3.9}$$

rather than zero. This implies an additional linear term in the pressure at low temperatures:

$$P^T\sim P^{T=0}+\frac{\zeta(3)}{8\pi a^3}T, \qquad aT\ll 1. \tag{3.10}$$

Exclusion of the TE zero mode will also reduce the linear temperature dependence expected at high temperatures,

$$P^T\sim -\frac{\zeta(3)}{8\pi a^3}T, \qquad aT\gg 1, \tag{3.11}$$

one-half the usual ideal metal result seen in (2.10a), and this is indeed predicted in numerical results [see figure 4 of [38] for $a>5\,\mu$m, for example.]

Most experiments are carried out between a sphere (of radius R) and a plane. In this circumstance, if $R\gg a$, a being the separation between the sphere and the plate at the closest point, the force may be obtained from the proximity force approximation,

$$\mathcal{F}=2\pi R F(a), \tag{3.12}$$

$F(a)$ being the free energy for the case of parallel plates separated by a distance a. Thus in the idealized description, the low temperature dependence including our linear term is

$$\mathcal{F} \sim -\frac{\pi^3 R}{360a^3}\left[1 - \frac{45}{\pi^3}\zeta(3)aT + \frac{360}{\pi^3}\zeta(3)(aT)^3 - 16(aT)^4\right], \qquad aT \ll 1 \tag{3.13}$$

Since this conversion is trivial, in the following we will restrict attention to the straightforward parallel plate situation.

These results are only approximate, because they assume the metal is ideal except for the exclusion of the TE zero. Elsewhere, we have referred to this model as the Modified Ideal Metal (MIM) model [38, 39]. Evidently, for sufficiently low temperatures the approximation used here, that $\zeta_1 \gg \gamma$, breaks down, the function $f(m)$ becomes continuous, and the linear term disappears. Indeed, numerical calculations based on real optical data for the permittivity show this transition. An example of such a calculation is presented in section 6. There, in figure 1, we see a negative slope in the quantity P/P_C as a function of the plate separation a in the region between 1 and 2 micrometers. This slope is approximately $-0.1/\mu$m. Here P is the pressure between the plates at 300 K, while P_C is the ideal Casimir pressure (2.5). If we compare this to our approximate prediction (3.10),

$$\frac{P^{T=300\,\mathrm{K}}}{P_C} \approx 1 - \frac{30}{7.62}\frac{\zeta(3)}{\pi^3}\frac{a}{\mu\mathrm{m}} = 1 - 0.15\frac{a}{\mu\mathrm{m}}, \tag{3.14}$$

the slope and intercepts agree at the 20% level. Accurate numerical results between real metal plates and sphere are given in [33].

Because this linear behavior does not persist at arbitrarily small temperatures, it is clear that the conflict with the third law anticipated in the arguments in the previous section do not apply. In fact, as we shall now see, the entropy does go to zero at zero temperature.

4. Arguments in favor and against the TE zero mode

As noted above, there are strong thermodynamic and electrodynamic arguments in favor of the exclusion of the TE zero mode. Essentially, the point is that a realistic physical system can have only one state of lowest energy. Electrodynamically, one can start from the Kramers-Kronig relation that relates the real and imaginary part of the permittivity, required by causality, which can be written in the form of a dispersion relation for the electric susceptibility [47]

$$\chi(\omega) = \frac{\omega_p^2}{4\pi}\int_0^\infty \mathrm{d}\omega' \frac{p(\omega')}{\omega'^2 - (\omega + \mathrm{i}\epsilon)^2}. \tag{4.1}$$

If the spectral function $p(\omega') \geq 0$ is nonsingular at the origin, it is easily seen that $\omega^2\chi(\omega) \to 0$ as $\omega \to 0$, which as shown in the previous section implies the absence of the TE zero mode. Conversely, $p(\omega')$ must have a δ-function singularity at the origin to negate this conclusion. This would seem implausible for any but an overly idealized model. In contrast, in the Drude model

$$p(\omega') = \frac{2}{\pi}\frac{\gamma}{\omega'^2+\gamma^2} \to 2\delta(\omega') \qquad \gamma \to 0. \tag{4.2}$$

It has been objected that rather then employing bulk permittivities as done in the usual expression for the Lifshitz formula, one should use surface impedances instead [48, 49, 34, 30]. Indeed this may be done, but it leads to identical results. The surface impedance merely expresses the linear relation between tangential components of the electric and magnetic fields at the interface between the two media,

$$\mathbf{E}_\perp = Z(\omega, \mathbf{k}_\perp)\mathbf{B}_\perp \times \mathbf{n}, \qquad \mathbf{n} \times \mathbf{E}_\perp = Z(\omega, \mathbf{k}_\perp)\mathbf{B}_\perp, \tag{4.3}$$

where $\mathbf{n}$ is the normal to the interface at the point in question. From Maxwell's equations we deduce [47, 39] for the reflection coefficient for the TE modes

$$r^{\mathrm{TE}} = -\frac{\zeta + Z\kappa}{\zeta - Z\kappa}, \qquad \kappa^2 = \zeta^2 + k_\perp^2, \tag{4.4}$$

and the surface impedance is§

$$Z = -\frac{\zeta}{\sqrt{\zeta^2[\varepsilon(\mathrm{i}\zeta)-1]+\kappa^2}}. \tag{4.5}$$

From this reflection coefficient the Lifshitz formula is constructed according to $d = \left(r^{\mathrm{TE}}\right)^{-2} \mathrm{e}^{2\kappa a} - 1$. Evidently the resultant expression for the Lifshitz pressure coincides with that found from the permittivity, seen for example in (3.6). This coincidence has been well recognized by previous authors [51, 52]. The reason why Mostepanenko and collaborators obtain a different result is that they omit the transverse momentum dependence in (4.5) and thereby argue that at zero frequency Z vanishes,

$$Z \to -\frac{1}{\sqrt{\varepsilon(\mathrm{i}\zeta)}} \sim \frac{\sqrt{\gamma}}{\omega_p}\sqrt{\zeta}, \tag{4.6}$$

which is the content of the normal skin effect formula

$$Z(\omega) = -(1-\mathrm{i})\sqrt{\frac{\omega}{8\pi\sigma}} \tag{4.7}$$

where σ is the conductivity. [These two formulas are seen to be identical if we replace $\omega = \mathrm{i}\zeta$ and recognize that $\gamma = \omega_p^2/(4\pi\sigma)$.] These formulas apply when we have the restriction appropriate to real photons $k_\perp^2 \leq \omega^2$. However, no such mass-shell condition applies to the virtual or evanescent photons involved in the thermal Casimir effect. The same sort of error seems to be made by Torgerson and Lamoreaux [53, 54], and by Bimonte [55, 56].

As noted above, use of the plasma model in the reflection coefficients would lead to the conventional temperature dependence, but this dispersion relation is inconsistent with real data. However, it has been argued that in the ideal Bloch-Grüneisen model

§ Here we have assumed that the permittivity is independent of transverse momentum. In principle this is incorrect, although optical data suggest that the transverse momentum dependence of ε is rather small. See also [50].

[57] the relaxation parameter goes to zero at zero temperature. However, real metals exhibit scattering by impurities; in any case, at sufficiently low temperatures the residual value of the relaxation parameter does not play a role, as the frequency characteristic of the anomalous skin effect becomes dominant [58]. Moreover, the authors of [30, 34] also extrapolate the plasma formula from the infrared region down to zero frequency, whereas in fact frequencies very small compared to the frequency corresponding to the separation distance play a dominant role in the temperature dependence [58]. Finally, we emphasize that all present experiments are carried out at room temperature, where the known room temperature data are relevant.

The principal reason for the theoretical controversy has to do with the purported violation of the third law of thermodynamics if the TE zero mode is not included. If ideal metal reflection coefficients are used otherwise (the MIM model) such a violation indeed occurs, because the free energy per unit area for small temperature then behaves like

$$F = F_0 + T\frac{\zeta(3)}{16\pi a^2}. \tag{4.8}$$

However, we and others have shown [59, 39, 58, 60] that for real metals, the free energy per area has a vanishing slope at the origin. Indeed, in the Drude model we have

$$F = F_0 + T^2\frac{\omega_p^2}{48\gamma}(2\ln 2 - 1), \tag{4.9}$$

for sufficiently low temperatures. There is, however, an intermediate range of temperatures where it is expected that the entropy is negative. We do not believe that this presents a thermodynamic difficulty, and reflects the fact that the electrodynamic fluctuations being considered represent only part of the complete physical system [59, 38, 33], although this is not a universal opinion [58]. New calculations are underway, showing explicitly the zero slope of the curve for the free energy near $T = 0$, thus corresponding to zero entropy [61].

Two other recent papers also lend support to our point of view. Jancovici and Šamaj [62] and Buenzli and Martin [63] have examined the Casimir force between ideal-conductor walls with emphasis on the high-temperature limit. Not surprisingly, ideal inert boundary conditions are shown to be inadequate, and fluctuations within the walls, modeled by the classical Debye-Hückel theory, determine the high temperature behavior. The linear in temperature behavior of the Casimir force is found to be reduced by a factor of two from the behavior predicted by an ideal metal, just as in (3.11). This is precisely the signal of the omission of the $m = 0$ TE mode. Thus, it is very hard to see how the corresponding modification of the low-temperature behavior can be avoided.

Further support for our conclusions can be found in the recent paper of Sernelius [64], who calculates the van der Waals-Casimir force between gold plates using the Lindhard or random phase approximation dielectric function. Spatial dispersion plays a crucial role in his calculations. For large separation, the force is one-half that of the ideal metal, just as in the calculations in [62, 63]. For arbitrary separations between the plates, Sernelius' results numerically nearly exactly coincide with his earlier ones [65, 42] where dissipation (i.e., nonzero relaxation parameter) is included but no spatial dispersion. In his new calculation, the inclusion of dissipation has negligible additional effect. His new results thus essentially coincide with ours. There are actually physical reasons why we should expect spatial dispersion not to be essential for the temperature correction to the Casimir force. Namely, this nonlocal effect is connected with the anomalous skin effect, which comes into play when the mean free path is much larger than the penetration depth. The phenomenon occurs for metals at very low temperatures, or at very high frequencies, and should not be of importance here. The condition for the argument is, however, that the usual Fresnel boundary conditions remain valid.

5. Experimental constraints

We have marshaled theoretical arguments that seem to us quite overwhelming in favor of the absence of the TE zero mode in the temperature dependence of the Casimir force between real metal plates, which seem to imply unambiguously that there should be large ($\sim 15\%$) thermal corrections to the Casimir force at separations of order 1 micrometer. New detailed calculations based on this theory, and using real optical data for aluminum, are discussed in the following section. The difficulty is that, experimentally, it is not easy to perform Casimir force measurements at other than room temperature, so current constraints on the theory all come from room temperature experiments. Then all one can do is compare the theory at room temperature with the experimental results, which must be corrected for a variety of effects, such as surface roughness, finite conductivity, and patch potentials. A deviation between the corrected zero temperature theory and the room temperature observations then is taken as a measure of the temperature correction.

The temperature correction is evidently relatively largest at the largest separations, where, unfortunately, the total Casimir force is weakest. Lamoreaux's early experiments [9] were conducted at the 1 μm scale, so if they were accurate to 10% they would have seen the effect our theory predicts, but probably, in spite of Lamoreaux's assertion, they were not so accurate, because few essential corrections were included [66]. The experiments of Mohideen *et al* [13, 14, 15] were much more accurate, but because they were conducted at much smaller distances, even our rather large temperature correction would have remained inaccessible. It is the most recent experiments of the Purdue group [29, 30] that claim the extraordinarily high precision to be able to see our effect at distances as small as 100 nm. Indeed, they see no deviation from the corrected zero-temperature Lifshitz theory using optical permittivities, and hence assert that our theory is decisively ruled out. The effect we predict for the temperature correction is only 1.5% at a distance of 160 nm [35], so the measurement must be performed at the 1% level to see the effect there. (For the usually employed sphere-plate configuration, $\Delta\mathcal{F}/\mathcal{F} \approx 2.5\%$ at $a = 160$ nm.) Although they claim this degree of accuracy, it is doubtful that they have achieved it, because, for example, to achieve 1% accuracy, the separation would have to be determined to better than 0.3%, or 0.5 nm at $a = 160$ nm. Since the roughness in the surfaces involved is much larger than this (see also [67]), and

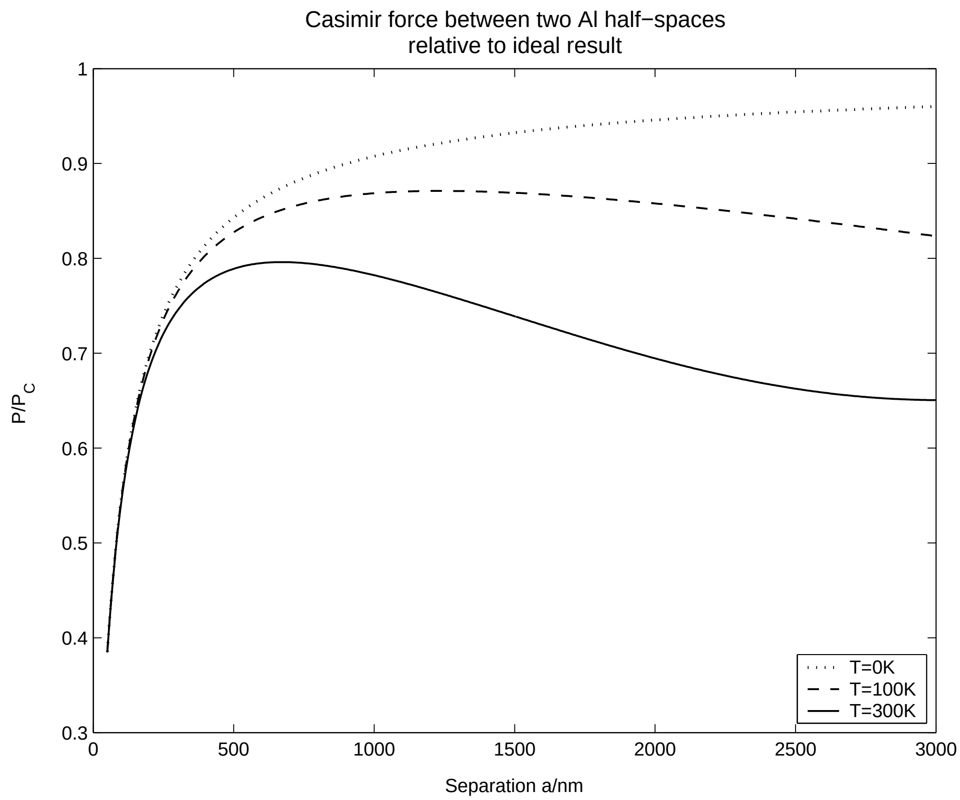

Figure 1. Temperature dependence of the Casimir force between aluminum plates.

other corrections (such as the fact that the metallic surfaces are actually thin films, and the effects of surface plasmon [68, 69]) have not been included, we have reason to be skeptical of such claims [70].

In any case, it would seem imperative to perform experiments at different temperatures in order to provide evidence for or against temperature dependence of Casimir forces. We understand such experiments are in progress. We encourage experimentalists to redouble their efforts to determine the presence or absence of such an effect in an unbiased manner, for the issues involved touch at the heart of our fundamental theoretical understanding of electrodynamics, statistical mechanics, and quantum field theory.

6. New calculations

To aid in the experimental disentanglement of this effect, we have carried out new calculations of the Casimir force between two infinite half-spaces made of aluminum, separated by a vacuum space of width a. (Other recent calculations appear in [35, 71, 46].) The results are shown in figure 1. (Figures 1–9 are taken from the Master's Thesis of SAE [72].) Formulas made use of are read off from (2.2) and are now given in usual dimensional units:

$$P_{T=0}(a) = -\frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\zeta \int_0^\infty \mathrm{d}k_\perp k_\perp \kappa_0 \left(\frac{\Delta_{\mathrm{TE}}^2 e^{-2\kappa_0 a}}{1-\Delta_{\mathrm{TE}}^2 \mathrm{e}^{-2\kappa_0 a}} + \frac{\Delta_{\mathrm{TM}}^2 e^{-2\kappa_0 a}}{1-\Delta_{\mathrm{TM}}^2 e^{-2\kappa_0 a}} \right), \tag{6.1a}$$

$$P_{T>0}(a) = -\frac{k_B T}{\pi}{\sum_{m=0}^{\infty}}' \int_0^\infty \mathrm{d}k_\perp k_\perp \kappa_0 \left(\frac{\Delta_{\mathrm{TE}}^2 e^{-2\kappa_0 a}}{1-\Delta_{\mathrm{TE}}^2 e^{-2\kappa_0 a}} + \frac{\Delta_{\mathrm{TM}}^2 e^{-2\kappa_0 a}}{1-\Delta_{\mathrm{TM}}^2 e^{-2\kappa_0 a}} \right), \tag{6.1b}$$

where a is the separation between plates (of infinite thickness), and

$$\Delta_{\mathrm{TE}} = \frac{\kappa-\kappa_0}{\kappa+\kappa_0}, \tag{6.2a}$$

$$\Delta_{\mathrm{TM}} = \frac{\kappa-\varepsilon(i\zeta)\kappa_0}{\kappa+\varepsilon(\mathrm{i}\zeta)\kappa_0}, \tag{6.2b}$$

$$\kappa = \sqrt{k_\perp^2 + \varepsilon(i\zeta)\frac{\zeta^2}{c^2}}, \tag{6.2c}$$

$$\kappa_0 = \sqrt{k_\perp^2 + \frac{\zeta^2}{c^2}}. \tag{6.2d}$$

Here, $\varepsilon(\mathrm{i}\zeta) = \epsilon(\mathrm{i}\zeta)/\epsilon_0$ is the usual permittivity relative to the vacuum. In the case of finite temperatures, $\zeta = \zeta_m = 2\pi m k_B T/\hbar$, and a standard Lifshitz substitution of integration variables was made during calculations (see for example (3.2b) of [38]). The results are plotted relative to the standard Casimir pressure P_C in (2.5). Calculations have been carried to a relative accuracy of better than 10^{-4}. Even at $T = 0$ there are large deviations from the ideal Casimir result at all distance scales.

To illustrate the contributions of the TE and TM modes, figures 2–4 depict the TE and TM integrands of a Casimir pressure expression of the type

$$P_{T=0} = \int_0^\infty \mathrm{d}\zeta \int_0^\infty \mathrm{d}k_\perp [I_{\mathrm{TE}}(\mathrm{i}\zeta, k_\perp) + I_{\mathrm{TM}}(\mathrm{i}\zeta, k_\perp)]. \tag{6.3}$$

It is clear that the TE term in the integrand falls off rapidly to zero as $\zeta \to 0$ whereas the TM term remains finite. The relative contributions to the pressure by the TE and TM modes are illustrated in figures 5–6. Evidently, the contribution of the TE mode rapidly decreases with increasing temperature and increasing plate separation.

6.1. Results for a five-layer model

Because many experiments have been carried out with a conducting surface between parallel capacitor plates it is useful to consider the five layer geometry which has been treated repeatedly by Tomaš in recent years [73]. The geometry is defined by figure 7. All three slabs are assumed to be made of aluminum. We assume one intermediate plate of width b, immersed in a cavity of total width $c = a + a' + b$. Between the central plate and the two outer semi-infinite media we assume a vacuum ($\varepsilon_g = 1$). The quantity h is defined as $h = c - b$. The quantity δ is the deviation of the center of the plate from the midline of the cavity. The Casimir pressure at zero and finite temperature is given by

$$P_{T=0}(\delta; b, c) = \frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\zeta \int_0^\infty \mathrm{d}k_\perp \sum_{q=\mathrm{TE}}^{\mathrm{TM}} I_q(\mathrm{i}\zeta, k_\perp; \delta, b, c) \tag{6.4a}$$

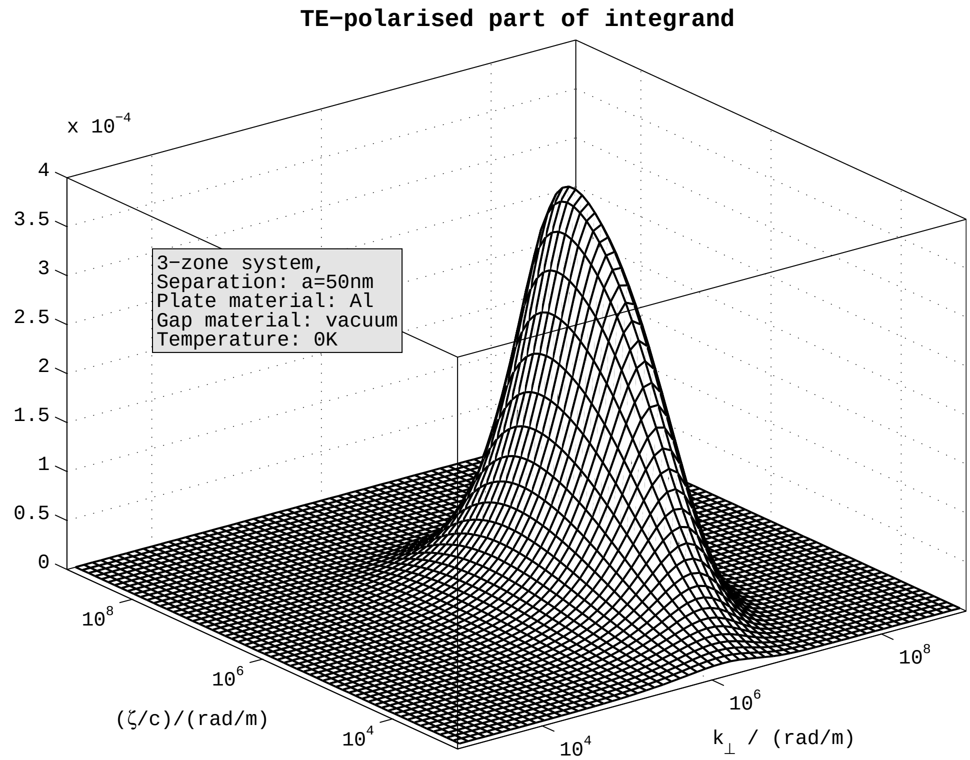

Figure 2. TE part of the integrand in (6.3).

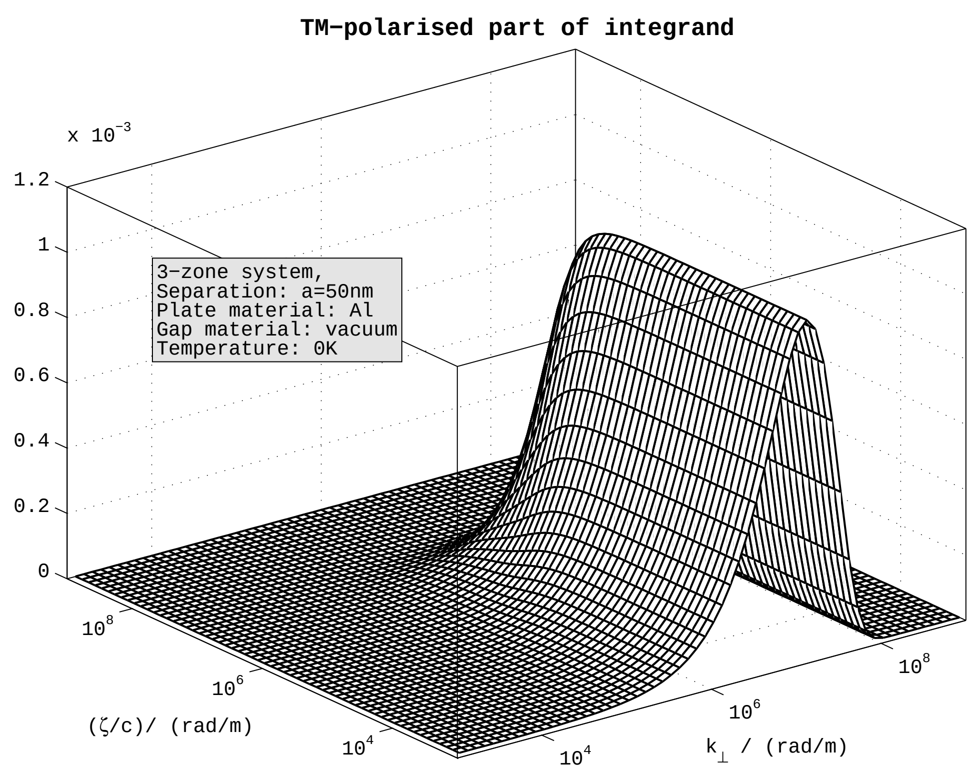

Figure 3. TM part of the integrand in (6.3).

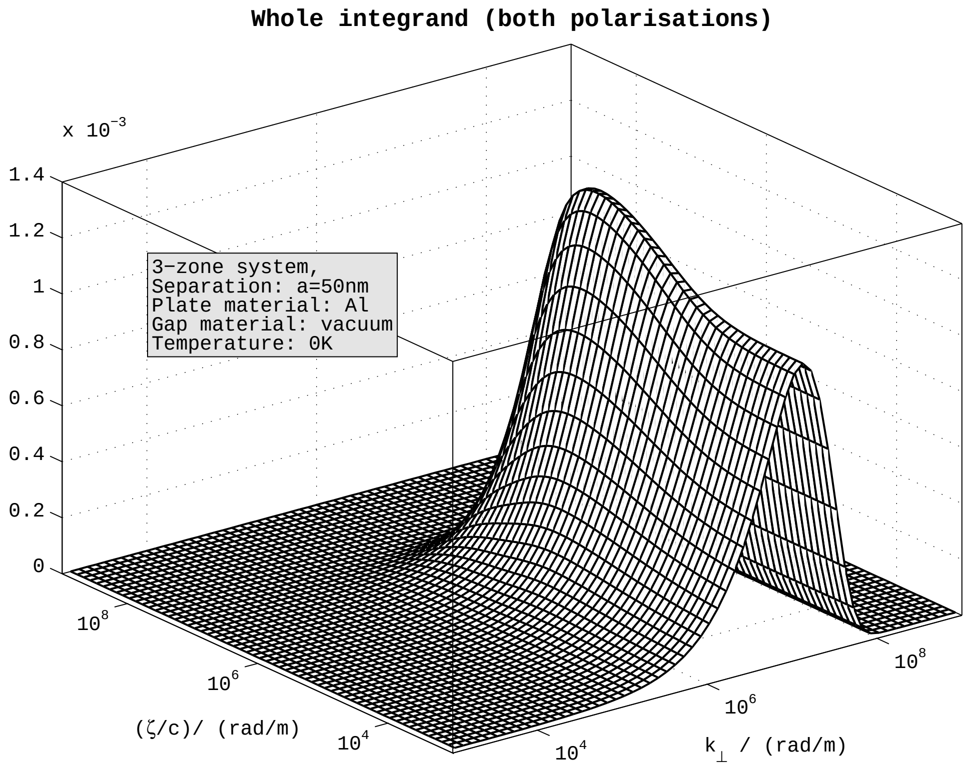

Figure 4. Total integrand in (6.3).

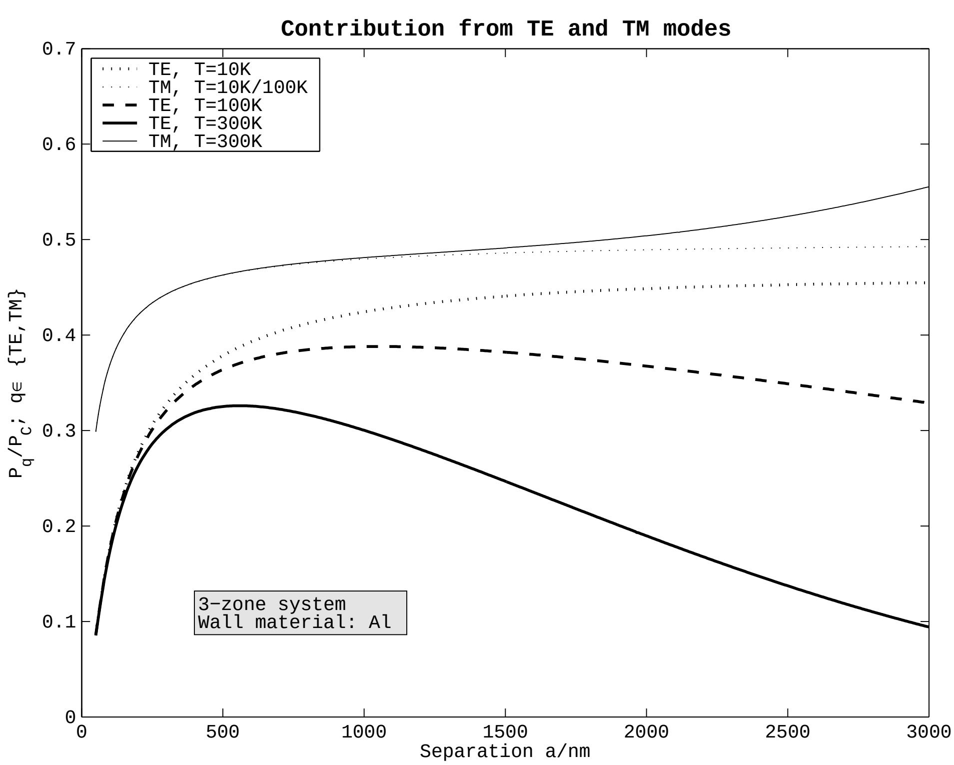

Figure 5. TE and TM contributions to the pressure shown in figure 1.

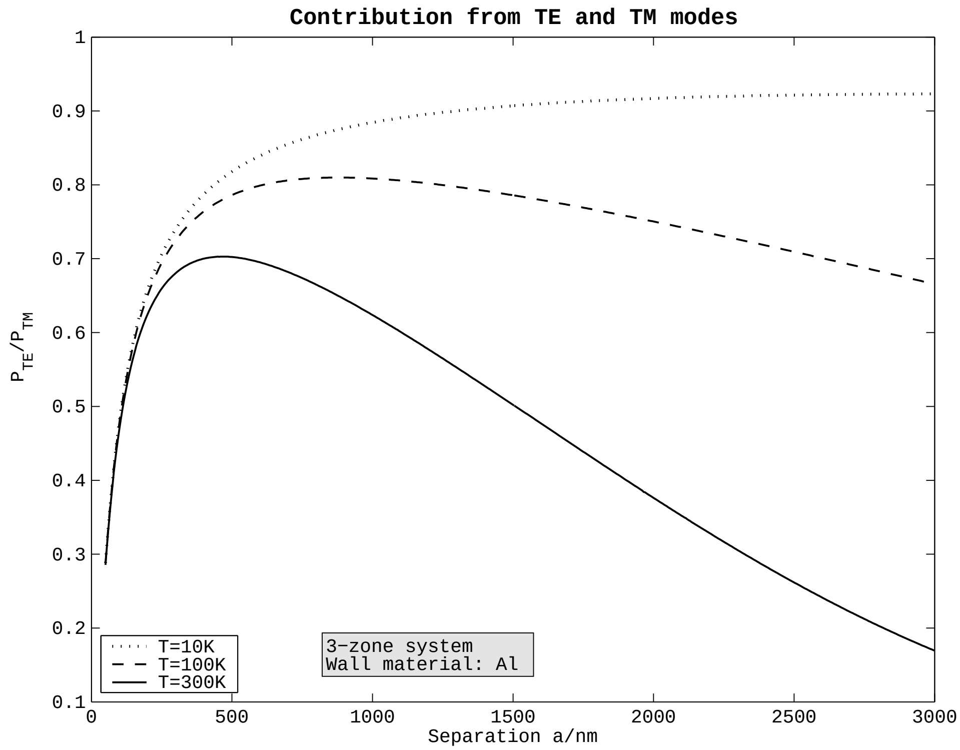

Figure 6. Ratio of the TE and TM mode pressures contributing to the pressure shown in figure 1.

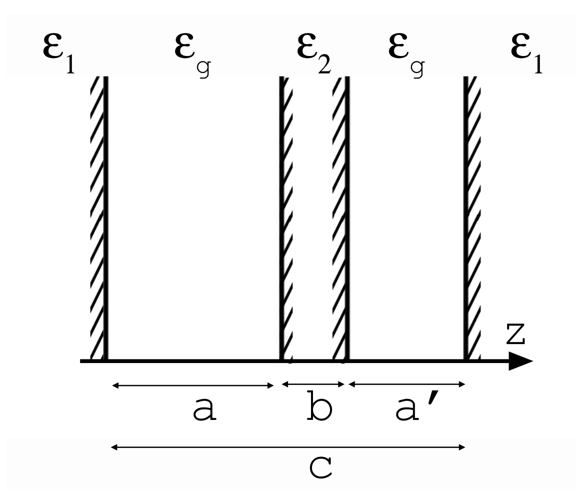

Figure 7. The 5-zone geometry. Here we have set $\varepsilon_g = 1$.

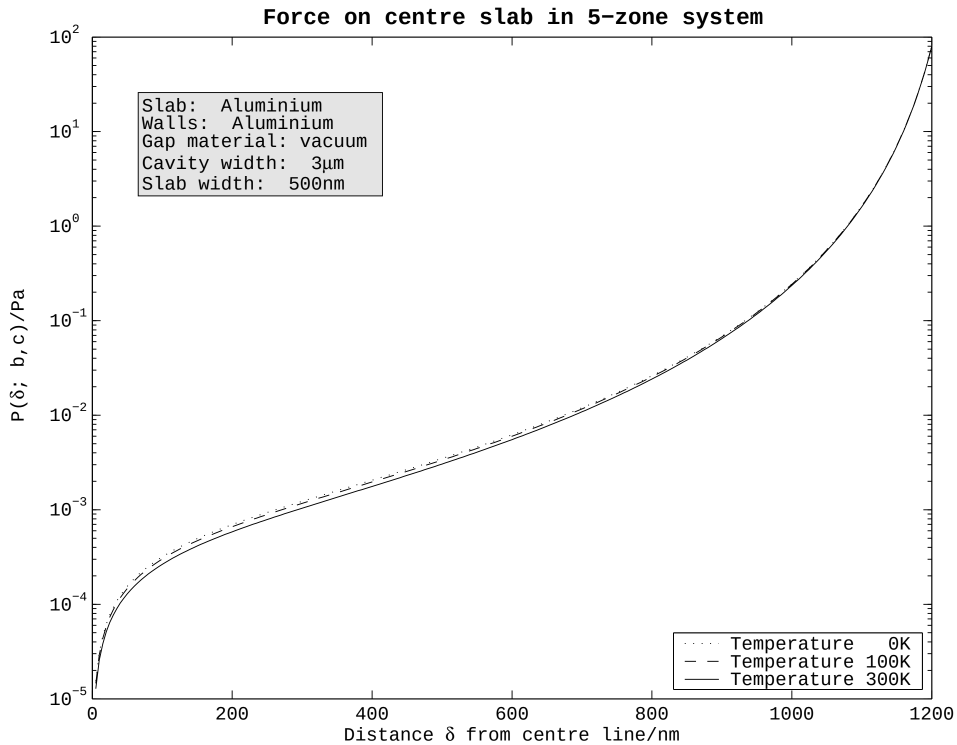

Figure 8. Pressure on the plate at a distance δ from the cavity center. If $\delta < 0$ one gets the antisymmetric prolongation about the position $\delta = 0$.

$$P_{T>0}(\delta; b, c) = \frac{k_B T}{\pi} {\sum_{m=0}^{\infty}}' \sum_{q=\mathrm{TE}}^{\mathrm{TM}} I_q(\mathrm{i}\zeta_m, k_\perp; \delta, b, c). \tag{6.4b}$$

Here the integrand is

$$\begin{aligned} I_q(\mathrm{i}\zeta, k_\perp; \delta, b, c) = k_\perp \kappa_0 2\Delta_{1q}\Delta_{2q}(1-\mathrm{e}^{-2\kappa_2 b})\mathrm{e}^{-\kappa_0 h}\sinh 2\kappa_0\delta \\ \times \left[-\Delta_{2q}^2 \mathrm{e}^{-2\kappa_2 b} + 1 - \Delta_{1q}^2 \mathrm{e}^{-2\kappa_0 h}(\mathrm{e}^{-2\kappa_2 b} - \Delta_{2q}^2) - 2\Delta_{1q}\Delta_{2q}(1-\mathrm{e}^{-2\kappa_2 b})\mathrm{e}^{-\kappa_0 h}\cosh 2\kappa_0 \delta\right]^{-1}, \end{aligned} \tag{6.5}$$

The Casimir force is positive for positive δ, and is antisymmetric around the cavity center $\delta = 0$. The index q in the Δ's in the formulas runs over the polarizations TE and TM, which are given by equations (6.2a), (6.2b).

Numerically, we choose $c = 3\,\mu$m and $b = 500$ nm. The pressure $P(\delta)$, calculated from $\delta = 0$ to $\delta = (c-b)/2 - 50$ nm, is shown in figure 8. All calculations are done with an accuracy of better that 10^{-4} in the final result, which should be sufficient for practical purposes. Figure 9 shows the pressure relative to Casimir's result for ideal conductors,

$$P_C = -\frac{\pi^2 \hbar c}{240}\left(\frac{1}{(h/2+\delta)^4} - \frac{1}{(h/2-\delta)^4}\right). \tag{6.6}$$

As for the finite temperature calculations, it has been checked that all terms in the sum (except for the zero mode) lie within the frequency domain covered by Lambrecht's

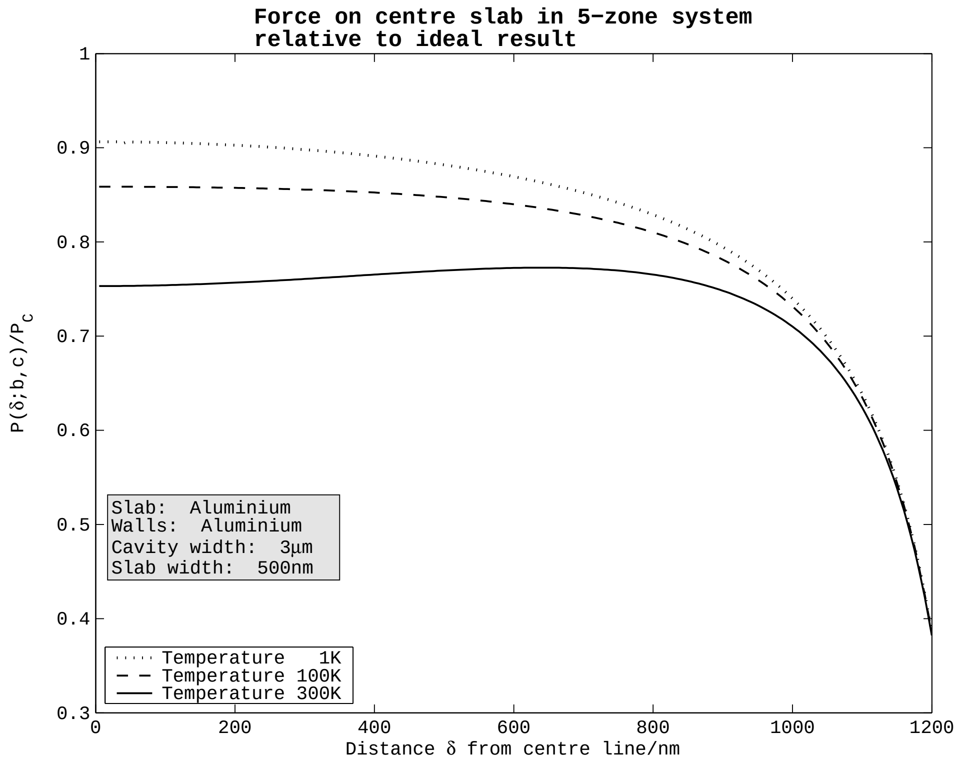

Figure 9. Pressure on the plate at a distance δ from the cavity center relative to Casimir's result (6.6) for ideal conductors. For $\delta < 0$ one gets the symmetric prolongation of the curve about the position $\delta = 0$.

data. There is thus no extra assumption made in the calculation, such as the property $\varepsilon\zeta^2/c^2 \to 0$ as $\zeta \to 0$, following from the Drude relation, except to ensure that there is no contribution from the TE zero mode. In the $T = 0$ case, the Drude relation is used for low frequencies. We believe that the contribution to the force coming from frequencies outside the Lambrecht region is very small. Again, we see large deviations from the ideal Casimir result at all temperatures, as well as relatively large temperature corrections, which we hope will be readily detectable in future experiments.

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