Dear Neil,

I have gone over your document of January 3 in which you give a “new” method of calculating the fluctuation. However, since you are considering a harmonic oscillator trap, and taking the classical limit, you must recover exactly the result I had earlier, with a spherically symmetric potential proportional to $r^2$, where $r$ is the distance to the center of the trap. Then, simply by dimensional analysis, the fluctuation, $\Delta \gamma^2$ must be proportional to $T$ not $T^2$.

Let me elaborate. You are assuming a partition function

$$Z = \sum_n e^{-\beta \hbar \omega (n_x+n_y+n_z)},$$

that is, that for a 3-dimensional harmonic oscillator. If $N = n_x + n_y + n_z$, the number of ways of getting a fixed value of $N$ is $\frac{1}{2}(N+1)(N+2)$, and so

$$Z = \sum_{N=0}^{\infty} \frac{1}{2}(N+1)(N+2)e^{-\beta \hbar \omega N}.$$

The sum may be easily carried out exactly. However, we are really only interested in the classical limit, for which $\hbar \to 0$, as you point out. Then $Z \propto \beta^{-3}$.

Now when we compute averages of the interaction energy, we may omit terms linear in the coordinates $x, y, z$, but not quadratic, by spherical symmetry. Then I easily reproduce your result for $\gamma_J$, modulo signs and sines. However, when one computes

$$\Delta \gamma^2 = \langle \gamma^2 \rangle - \langle \gamma \rangle^2$$

terms one dropped in computing $\langle \gamma \rangle$ now contribute, because they come in squared. So one encounters in $\Delta \gamma^2$ terms just like those found in $\langle \gamma \rangle$, for example

$$\langle z^2 \rangle = \langle \frac{1}{3} r^2 \rangle \propto \langle H \rangle$$

by the virial theorem. So the fluctuation has terms in it proportional to

$$\Delta \gamma^2 \propto -\frac{\partial}{\partial \beta} \ln Z \sim \frac{1}{\beta} = T.$$

This, of course, is just the result of the classical equipartition theorem.
So your result (8) cannot be correct. I think you must have neglected to square the terms which go away by symmetry in computing the mean. As I have been insisting, it appears that no matter what one does, the leading terms in the regime

\[ \hbar \omega \ll T \ll \tilde{T} \]

must be linear, not quadratic, in \( T \). This makes the experiment much harder to achieve.

Talk to you tomorrow,

Kim