The Bjorken Sum Rule in the Analytic Approach to Perturbative QCD

K.A. Milton[a,*], I.L. Solovtsov[a,b,†], O.P. Solovtsova[a,b,‡]

[a]Department of Physics and Astronomy, University of Oklahoma,
Norman, OK 73019 USA

[b]Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
141980 Dubna, Moscow Region, Russia

March 6, 1998

Abstract

Results of applying analytic perturbation theory (APT) to the Bjorken sum rule are presented. We study the third-order QCD correction within the analytic approach and investigate its renormalization scheme dependence. We demonstrate that, in the framework of the method, theoretical predictions of the Bjorken sum rule are, practically, scheme independent for the entire interval of momentum transfer.

PACS: 11.10.Hi; 11.55.Fv; 12.38.-t; 12.38.Bx

[*]E-mail: milton@mail.nhn.ou.edu

[†]E-mail: solovtso@thsun1.jinr.ru

[‡]E-mail: olsol@thsun1.jinr.ru

1 Introduction

A fundamental test of QCD is obtained by comparing the value of the quantum chromodynamics (QCD) coupling constant α_S extracted from experimental data at different energy scales with theoretical predictions given by the renormalization group method. The corresponding evolution law of $\alpha_S(Q^2)$ is now experimentally studied down to low momentum transfers, $Q^2 \sim 1\,\mathrm{GeV}^2$. The principal experimental information concerning α_S at low Q^2 comes from measurements of the inclusive hadronic τ-decay rate and from deep-inelastic scattering sum rules. As is well-known, theoretical QCD predictions for these processes are primarily based on perturbation theory (PT) improved by the renormalization group. However, the perturbative expansion is ill-defined at low energies and the conventional PT method of deriving the QCD running coupling constant leads to unphysical singularities of $\alpha_S(Q^2)$, such as a ghost pole, which are in conflict with the fundamental principle of causality. Higher-order PT corrections taken in the asymptotic form cannot resolve this problem and just add unphysical cuts. Moreover, at low values of the momentum transfer there is a strong dependence on the choice of renormalization scheme, which leads to essential ambiguities in the description of the physical quantity under consideration.

In [1], the conventional method of the renormalization group improvement of the PT approximation has been modified by requiring Källen-Lehmann analyticity, which reflects the principle of causality. As a result, the QCD running coupling constant has no unphysical singularities and at low energy scales behaves significantly differently than the conventional perturbative one. The method proposed in [1], called analytic perturbation theory (APT), gives a well-defined procedure of analytic continuation of the running coupling constant from the Euclidean (spacelike) into the Minkowskian (timelike) region [2, 3]. This fact allows one to describe the inclusive decay of the τ lepton in a self-consistent way [4], essential became the correct analytic properties are important in order to rewrite an initial integral expression for the R_τ-ratio in the form of a contour integral representation.

The renormalization scheme (RS) ambiguity which appears due to the truncation of the perturbative expansion represents a serious difficulty in carrying out perturbative calculations. There is no definite solution to this problem apart from computing indefinitely many terms in the perturbative expansions. To somehow avoid this difficulty, various optimization procedures have been applied, for example, the principle of minimal sensitivity [5] or the effective charge approach [6]. The RS dependence can be also reduced by using the Padé summation method [7]. As it has been argued in [8, 9], besides considering PT predictions in some preferred scheme, one should also study the stability of those predictions by varying parameters that define the RS over some acceptable interval. In the framework of the analytic approach the problem of the RS dependence has been studied for the process of e^+e^- annihilation into hadrons and the process of inclusive τ-decay in [10, 11], where it has been demonstrated that the APT approach can reduce the RS dependence drastically.

Apart from the τ decay, there is another important observable, the Bjorken sum rule [12], which allows one to extract α_S at low Q^2. In this paper we analyze the Bjorken sum rule, which is the integral of the difference between g_1 for the proton and neutron, in the framework of the APT method. We compare our result with standard PT predictions (see, e.g. [13]) and investigate the RS dependence of the various theoretical predictions.

2 The Bjorken sum rule within the analytic approach

The polarized Bjorken sum rule refers to the integral over all x at fixed Q^2 of the difference between polarized structure functions of the proton $g_1^{\rm p}$ and the neutron $g_1^{\rm n}$,

$$\Gamma_1^{\rm p-n}(Q^2) \;=\; \int_0^1 \; \left[g_1^{\rm p}(x,Q^2) - g_1^{\rm n}(x,Q^2)\right] dx \,. \tag{1}$$

The Bjorken integral (1) can be written in terms of the QCD correction $\Delta_{\rm Bj}$

$$\Gamma_1^{\rm p-n}(Q^2) \;=\; \frac{1}{6}\left|\frac{g_A}{g_V}\right| \, \left[1 \,-\, \Delta_{\rm Bj}(Q^2)\right] \,. \tag{2}$$

The value of the nucleon beta decay constant is taken to be $|g_A/g_V| = 1.2601 \pm 0.0025$ [14].

The perturbative QCD correction to the Bjorken sum rule in the three-loop approximation with the use of the $\overline{\rm MS}$ renormalization scheme and in the massless quark limit has the form

$$\Delta_{\rm Bj}^{\rm PT}(Q^2) \;=\; \frac{\alpha_{\rm PT}(Q^2)}{\pi} \;+\; d_1\left[\frac{\alpha_{\rm PT}(Q^2)}{\pi}\right]^2 \;+\; d_2\left[\frac{\alpha_{\rm PT}(Q^2)}{\pi}\right]^3 \,, \tag{3}$$

where for three active quarks the coefficients are $d_1^{\overline{\rm MS}} = 3.5833$ and $d_2^{\overline{\rm MS}} = 20.2153$ [15]. The perturbative running coupling constant $\alpha_{\rm PT}(Q^2)$ is obtained by integration of the renormalization group equation with the three-loop β-function.

As it has been demonstrated in [16] by using the Deser-Gilbert-Sudershan representation for the virtual forward Compton amplitude, the moments of the structure functions are analytic functions of Q^2 in the complex Q^2-plane with a cut along the negative part of the real axis. It is clear that the perturbative representation (3) violates these analytic properties due to the unphysical singularities of the PT running coupling constant for $Q^2 > 0$. To avoid this problem we apply the APT method, which gives the possibility of combining the renormalization group resummation with correct analytical properties of the QCD correction to the Bjorken sum rule.

Let us write down the QCD correction in the form of a spectral representation

$$\Delta_{\mathrm{Bj}}(Q^2) \;=\; \frac{1}{\pi}\int_0^\infty \frac{d\sigma}{\sigma + Q^2}\,\varrho(\sigma)\,, \tag{4}$$

where we have introduced the spectral function, which is defined as the discontinuity of $\Delta_{\mathrm{Bj}}(Q^2)$: $\varrho(\sigma) \;=\; \mathrm{Disc}\left\{\Delta_{\mathrm{Bj}}(-\sigma - \mathrm{i}\epsilon)\right\}/2\mathrm{i}\,.$

By calculating the spectral function $\varrho(\sigma)$ perturbatively we get a expression for $\Delta_{\mathrm{Bj}}(Q^2)$ which has the correct analytic properties and therefore has no unphysical singularities. For instance, the APT running coupling constant in the one-loop approximation has two terms:

$$\alpha_{\mathrm{APT}}(Q^2) \;=\; \frac{4\pi}{\beta_0}\left[\frac{1}{\ln\left(Q^2/\Lambda^2\right)} + \frac{1}{1 - Q^2/\Lambda^2}\right]\,. \tag{5}$$

Obviously, the first term in Eq. (5) has the standard PT form, but the second term (which appears automatically from the spectral representation and restores the correct analytic properties of the running coupling constant) has an essentially nonperturbative nature. If we rewrite the second term in Eq. (5) in terms of the PT coupling constant, we obtain an expression which has an essential singularity like $\exp\left(-4\pi/\beta_0\alpha_{\mathrm{PT}}\right)$ as $\alpha_{\mathrm{PT}} \to 0$. Therefore, the second term in Eq. (5) does not contribute to the conventional perturbative expansion. It has been argued in [1] that a similar situation holds also for the running coupling constant in higher order approximations. The asymptotic PT expression for the running coupling constant is an expansion in the small parameter $1/\ln(Q^2/\Lambda^2)$. This approximation violates the Q^2-analyticity of the running coupling constant and does not allow one to describe low energy scales – the perturbative series diverges in the infrared region. The APT method removes this difficulty and leads to a quite stable result for the entire interval of momentum. The difference between the shapes of the PT and APT running coupling constants becomes significant at low Q^2-scales.

It is convenient to write the three-loop APT approximation to $\Delta_{\rm Bj}(Q^2)$ as follows

$$\Delta_{\rm Bj}^{\rm APT}(Q^2) \;=\; \delta_{\rm APT}^{(1)}(Q^2) \,+\, d_1 \delta_{\rm APT}^{(2)}(Q^2) \,+\, d_2 \delta_{\rm APT}^{(3)}(Q^2)\,, \tag{6}$$

where the coefficients d_1 and d_2 are the same as in Eq. (3) and the functions $\delta_{\rm APT}^{(k)}(Q^2)$ are derived from the spectral representation and correspond to the discontinuity of the k-th power of the PT running coupling constant

$$\delta_{\rm APT}^{(k)}(Q^2) \;=\; \frac{1}{\pi^k}\int_0^\infty \frac{d\sigma}{\sigma \,+\, Q^2}\,{\rm Im}\,\left\{\alpha_{\rm PT}^k(-\sigma - {\rm i}\varepsilon)\right\}\,. \tag{7}$$

The function $\delta_{\rm APT}^{(1)}(Q^2)$ defines the APT running coupling constant, $\alpha_{\rm APT}(Q^2) \;=\; \pi\delta_{\rm APT}^{(1)}(Q^2)\,,$ which in the one-loop order is given by Eq. (5).

In the case of PT the QCD correction to the Bjorken sum rule is represented in the form of a power series in $\alpha_{\rm PT}$ [see Eq. (3)], but in the case of APT the same QCD correction is not a polynomial in the APT running coupling constant. As follows from Eq. (6), the first term of the expansion is $\alpha_{\rm APT}/\pi$, but the following terms are not representable as powers of $\alpha_{\rm APT}$.

Table 1: Terms of PT and APT series for the QCD correction to the Bjorken sum rule.

$\Gamma_1^{\rm p-n}(3\,{\rm GeV}^2) \;=\; 0.160$						
		$1st$-term		$2nd$-term		$3rd$-term
$\Delta_{\rm Bj}^{\rm PT}$	=	0.131	+	0.062	+	0.045
$\Delta_{\rm Bj}^{\rm APT}$	=	0.190	+	0.045	+	0.003

To illustrate the difference between the convergence properties of the PT expansion (3) and the APT series (6) we use, as an example, a typical value of $\Gamma_1^{\rm p-n} \;=\; 0.160 \pm 0.014$

at $Q^2 = 3\,\mathrm{GeV}^2$ taken from [7], fixing in such a way the value of the QCD correction in Eq. (2). In Table 1 we present numerical results for contributions to Δ_{Bj} in different orders for both the PT and the APT methods. From this table one can see that the higher order corrections to the Bjorken sum rule play a different role in the PT and APT approaches. The convergence of the APT series seems to be much more well behaved than is that of the PT expansion at such small $Q \simeq 1.73\,\mathrm{GeV}$.

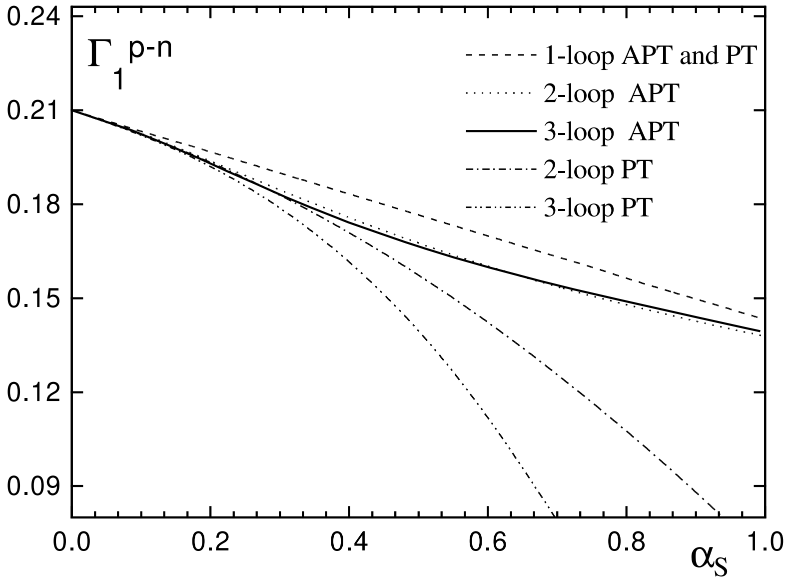

Figure 1: $\Gamma_1^{\mathrm{p-n}}$ *with* 1-, 2-, *and* 3-*loop QCD corrections vs. the coupling constant.*

Next in Fig. 1, we show $\Gamma_1^{\mathrm{p-n}}$ as a function of the QCD running coupling constant α_S in the PT and APT approaches. As outlined above in the PT case, the function $\Gamma_1^{\mathrm{p-n}}$ is an explicit function of the PT running coupling constant and in the one-loop approximation is represented by a straight line in Fig. 1, as a parabola in the two-loop case, and as a cubic curve in the three-loop one. At sufficiently large values of $\alpha_S \sim 0.5$, the difference between the 1-, 2-, and 3-loop PT predictions becomes very large. In the case of APT, the function $\Gamma_1^{\mathrm{p-n}}$ only in one-loop approximation is an explicit function of the APT running coupling constant. Of course, the coincidence of the one-loop PT and APT curves in Fig. 1 does not mean that the PT and APT approaches are physically identical, this is simply a matter of the linear form of the one-loop approximation – because the behavior of the PT and APT running coupling constants are rather different [see Eq. (5)]. The contribution of the higher loop corrections in the APT case is not so large as in the PT one and the corresponding curves in Fig. 1 are quite close to the linear function. Fig. 1 demonstrates that the APT result is more stable with respect to higher loop contributions.

Table 2: The QCD parameters extracted from different experimental inputs.

Input [Ref.]	Experiment		QCD parameters			
$Q_0^2 = 10\,\mathrm{GeV}^2$	$\Gamma_1^{\mathrm{p-n}}(Q_0^2)$	$\Delta_{\mathrm{Bj}}(Q_0^2)$	Λ_{PT}	$\alpha_{\mathrm{PT}}(Q_0^2)$	Λ_{APT}	$\alpha_{\mathrm{APT}}(Q_0^2)$
(a) [17]	0.183 ± 0.034	0.129 ∓ 0.162	467 MeV	0.275	741 MeV	0.301
(b) [18]	0.195 ± 0.029	0.072 ∓ 0.138	138 MeV	0.177	149 MeV	0.179

To compare the Q^2-evolution of the Bjorken sum rule in the APT and PT approaches, we use as input new experimental values of $\Gamma_1^{\mathrm{p-n}}(Q_0^2 = 10\,\mathrm{GeV}^2)$ given by the SMC Collaboration in Refs. [17] and [18]. These values are presented in Table 2 as input (a) and (b). The experimental errors of the present data are too large to determine the QCD correction. However, to illustrate the evolution, we fix the values of the parameters Λ or the corresponding values of α_S in the PT and APT cases from the central values obtained from the data (a) and (b). By using these normalization points, we plot, in Fig. 2, the Q^2 dependence of the QCD correction Δ_{Bj}. Besides the normalization points, for illustration, we also represent in Fig. 2 the recent data of the E154 Collaboration [19] obtained for small Q^2. Normalizing at the point (b) we get small values of Λ which are close to each other in the PT and APT cases (see Table 2); therefore, the corresponding curves in Fig. 2 are

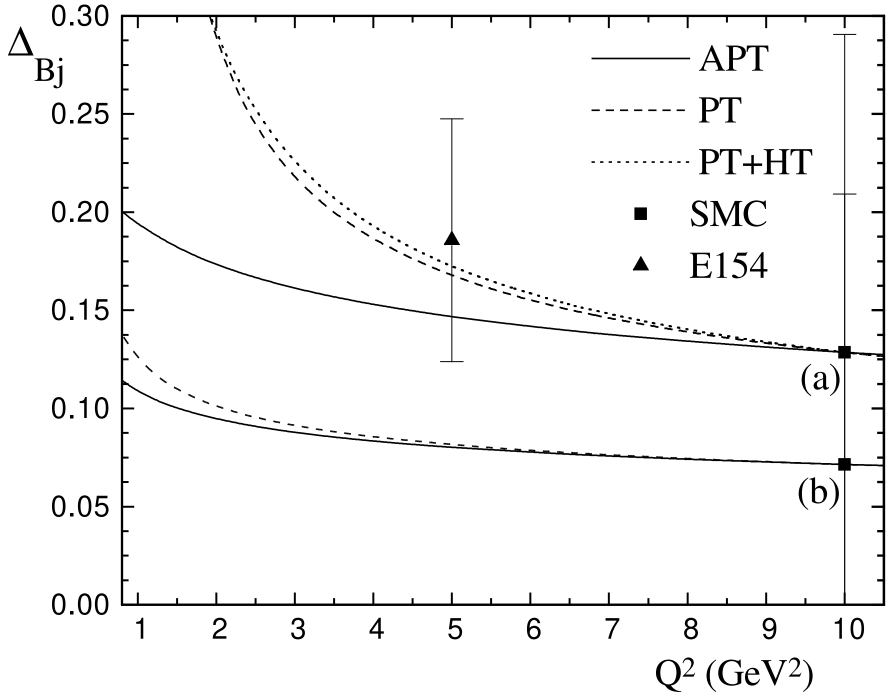

Figure 2: *The 3-loop QCD correction to Bjorken sum rule in the APT and PT approaches at different experimental normalization points.*

also close to each other. However, the values of the running coupling constant extracted in such a way are too small to have good agreement with other experimental data, for example, with the data of the E154 Collaboration plotted in Fig. 2 and with the value of α_S extracted from the semileptonic τ decay [14]. If the more realistic normalization at the point (a) is used, the difference between the PT and APT predictions becomes large. Now, we have the value $\Lambda_{\rm APT} = 741\,{\rm MeV}$ [see Table 2 input (a)] that is consistent with value extracted from τ decay [11], $\Lambda^{\tau}_{\rm APT} = 871 \pm 155\,{\rm MeV}$. The discrepancy of the PT value $\Lambda_{\rm PT} = 467\,{\rm MeV}$ and that obtained from the τ decay, $\Lambda^{\tau}_{\rm PT} = 385 \pm 27\,{\rm MeV}$, appears to be large.

The dotted curve in Fig. 2 demonstrates that the inclusion of the higher-twist (HT) corrections does not change the overall picture of the Q^2-dependence of the PT prediction in the interval under consideration. However, the higher-twist corrections do reduce the value of Λ. By using the higher-twist coefficient $c_{\rm HT} = -0.03\,{\rm GeV}^2$ (see discussion in [13]), we obtain the value of $\Lambda_{\rm PT} = 387\,{\rm MeV}$ which agrees well with the above value of $\Lambda^{\tau}_{\rm PT}$. Inclusion of such higher-twist corrections is, however, not necessary in the APT approach to achieve agreement between the QCD scale parameter Λ extracted from the Bjorken sum rule and τ decay. Unfortunately, the experimental errors on the Bjorken sum rule are too large to reach a definite conclusion.

3 Renormalization scheme dependence

A truncation of the perturbative expansion leads to some uncertainties in theoretical predictions for a physical quantity arising from the RS dependence of the partial sum of the series. At low momentum scales these uncertainties may become to be very large (see, for example, an analysis in [20]). Thus, it is not enough to obtain a result in some preferred scheme, but rather it is important to investigate its stability by varying the parameters that define the RS over some acceptable domain.

Consider the RS dependence of our results. The coefficients d_1 and d_2 in Eq. (3) are RS dependent. In the three-loop β-function

$$\beta(a) = \mu^2 \frac{\partial\, a}{\partial\, \mu^2} = -\beta_0\, a^2\, (1 + c_1 a + c_2 a^2)\,, \tag{8}$$

the coefficient c_2 also depends on RS.

In the framework of the conventional approach, there is no resolution of this problem of the RS dependence apart from calculating indefinitely many further terms in the PT expansion, and there is no fundamental principle upon which one can choose one or another preferable RS. However, it is possible to define a class of 'natural' RS's by using the so-called cancellation index criterion [9]. According to this criterion a class of 'well-behaved' RS's are defined a such a way that the degree of cancellation between the different terms in the second RS-invariant [5]

$$\rho_2 = d_2 + c_2 - d_1^2 - d_1\, c_1 \tag{9}$$

is not to be very large. The degree of these cancellations can be measured by the cancellation index [9]

$$C = \frac{1}{|\rho_2|}\left(|d_2| + |c_2| + d_1^2 + |d_1|\, c_1\right) . \tag{10}$$

By taking some maximal value of the cancellation index $C_{\max}$ one should investigate the stability of predictions for the RS's with $C \leq C_{\max}$. In the case of the $\overline{\mathrm{MS}}$-scheme, for the Bjorken sum rule, the value of the cancellation index is $C_{\overline{\mathrm{MS}}} = 8$. We will consider this value as a boundary for the class of 'natural' schemes.

Three RS dependent parameters d_1, d_2 and c_2 are connected by the second RS-invariant (9), and, therefore, any RS is defined by the pair of numbers (d_1, c_2). In Fig. 3 we plot the PT predictions represented by dashed lines for three schemes: $\overline{\mathrm{MS}}$; A with parameters $d_1^{\mathrm{A}} = -4.3$, $c_2^{\mathrm{A}} = 0$, and with the value of cancellation index $C_{\mathrm{A}} = 8$; and B with parameters $d_1^{\mathrm{B}} = 0$, $c_2^{\mathrm{B}} = 14.5$, and $C_{\mathrm{B}} = 4.3$. As a example of an optimal scheme, we consider the effective charge (ECH) approach in which $d_1^{\mathrm{ECH}} = d_2^{\mathrm{ECH}} = 0$, $c_2^{\mathrm{ECH}} = \rho_2$, and $C_{\mathrm{ECH}} = 1$. The dotted curve presents the ECH result. For the sake of illustration we also show the experimental data taken from [17, 19, 21].[1] To normalize all curves, we used the experimental value for Δ_{Bj} at $Q_0^2 = 10\,\mathrm{GeV}^2$ given in Table 2 as input (a).[2] The description of experimental data

[1]If one returns to Fig. 1 and compares the properties of convergence of the PT and APT series for the data [21], where $\Gamma_1^{\mathrm{p-n}}(Q^2 \simeq 1\,\mathrm{GeV}^2) \simeq 0.1$, we can see that the PT series is not stable with respect to higher loop corrections.

[2]To this end, we used $C < 8$ for the B-scheme in order to satisfy the normalization condition.

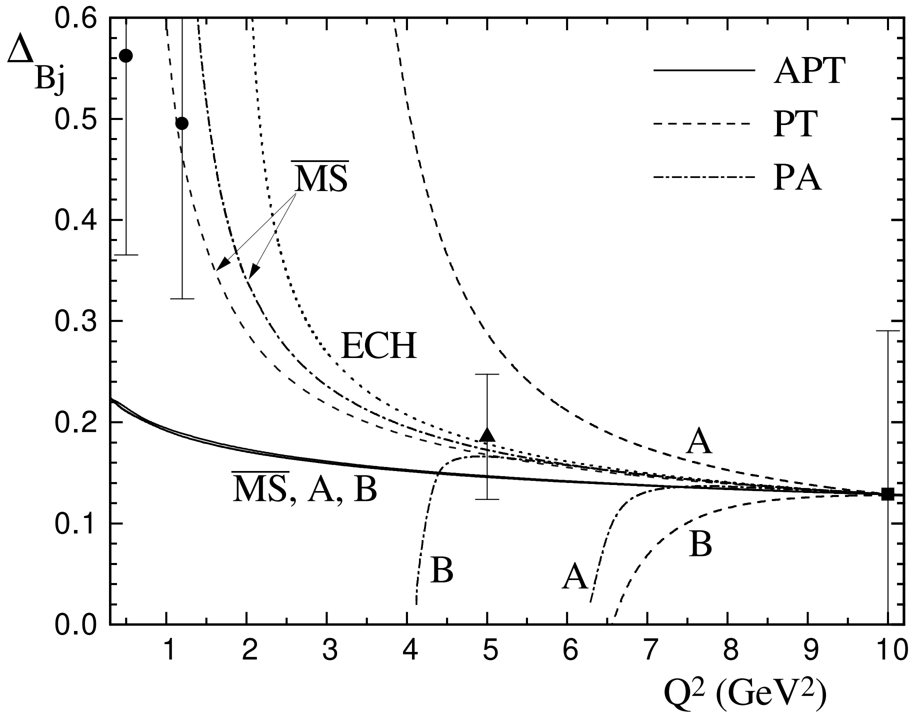

Figure 3: *Renormalization scheme dependence of predictions for* $\Delta_{\rm Bj}$ *vs.* Q^2 *for the APT and PT expansions. The curves correspond to the APT approach (solid), the PT evolution in* $\overline{MS}$*, A and B schemes (dash), and in ECH (dot); the PA result in* $\overline{MS}$*, A, B schemes (dash-dot). The SMC data [17] is denoted by a square, the triangle is the E154 data [19], and circles are E143 data [21].*

within the $\overline{\mathrm{MS}}$ scheme seems to be quite good; however, as has been mentioned above, there is no reason why the $\overline{\mathrm{MS}}$ scheme is preferable over, say, the A-scheme. The corresponding series have the forms $\Delta_{\mathrm{Bj}}^{\overline{\mathrm{MS}}} = x + 3.583x^2 + 20.215x^3$ and $\Delta_{\mathrm{Bj}}^{\mathrm{A}} = x - 4.36x^2 + 16.834x^3$, where $x \equiv \alpha_S(Q^2)/\pi$.

Thus the uncertainties coming from the RS dependence of perturbative calculations are rather large. At the same time, the APT predictions (solid curves) practically coincide with each other and, therefore, are RS independent. This stability reflects the existence of a universal limiting value [1] of the APT running coupling constant and the small values of higher loop corrections for the entire interval of momentum. We evaluated also $\Delta_{\mathrm{Bj}}(Q^2)$ using the Padé approximant (PA) [0/2] of the PT series (see [7]). The results are shown as three dash-dot curves corresponding to the $\overline{\mathrm{MS}}$, A and B schemes. The PA improves the stability properties, but the sensitivity to the choice of RS becomes very large for small momentum, $Q^2 < 5$ GeV^2.

Thus the conventional PT prediction at small momentum transfers has a very large RS ambiguity. The APT approach reduces the RS dependence drastically. At the three loop level the APT result is, practically, RS independent.

4 Summary and conclusion

We have considered the Bjorken sum rule by using the APT approach and have demonstrated that the convergence properties of the APT are much better than are those of the PT expansion. The APT results have extraordinary stability with respect to higher loop corrections and also to the choice of the RS. The analysis performed shows a quite different Q^2 evolution of the Bjorken sum rule in the PT and APT descriptions. At low Q^2 of order a few GeV^2, the conventional PT approach leads to a very rapidly changing function for the Bjorken integral in many RS's. At the same time, the three-loop APT prediction is practically RS independent and the Q^2 evolution is described by a slowly changing function. Unfortunately, the present experimental data for the Bjorken sum rule have large errors, which does not allow us to discriminate between the approaches experimentally. Nevertheless note that experimental data of the E143 Collaboration, which have just appeared [22], show, practically, the same value of $\Gamma_1^{\rm p-n}$ at $Q^2 = 2$, 3 and $5\,{\rm GeV}^2$, which agrees well with the slow APT Q^2 evolution.

The APT approach incorporates "perturbative" power corrections to secure the required analytic properties of the running coupling constant. These corrections come from the perturbative short distance analysis and their appearance is not inconsistent with the operator product expansion [23]. In this note we have concentrated on the analytically improved perturbative contribution to the Bjorken sum rule and did not consider higher-twist effects. The higher-twist terms can, in principle, be potentially important; however, at this stage of the analysis they lead to an additional uncertainty because the corresponding values of parameters do not well determined [24]. Nevertheless, note here that in the framework of the analytic approach the phenomenological role of "non-perturbative" power corrections, which are controlled by the operator product expansion, is changed, and it will, in the future, be interesting to consider this problem further.

Acknowledgement

The authors would like to thank D.V. Shirkov for interest in this work and O. Nachtmann who brought the paper [16] to our attention. The authors would also like to express their sincere thanks to L. Gamberg for helpful discussions. Partial support of the work by the US National Science Foundation, grant PHY-9600421, by the US Department of Energy, grant DE-FG-02-95ER40923, by the University of Oklahoma, and by the RFBR, grant 96-02-16126 is gratefully acknowledged.

References

[1] D.V. Shirkov, I.L. Solovtsov, JINR Rap. Comm. No 2[76]-96 (1996) 5; hep-ph/9604363; Phys. Rev. Lett. 79 (1997) 1209.

[2] K.A. Milton, I.L. Solovtsov, Phys. Rev. D 55 (1997) 5295.

[3] K.A. Milton, O.P. Solovtsova, preprint OKHEP-97-06, hep-ph/9710316 (to be publ. in Phys. Rev. D).

[4] K.A. Milton, I.L. Solovtsov, O. P. Solovtsova, Phys. Lett. B 415 (1997) 104.

[5] P.M. Stevenson, Phys. Rev. D 23 (1981) 2916.

[6] G. Grunberg, Phys. Rev. D 29 (1984) 2315.

[7] J. Ellis, E. Gardi, M. Karliner, M. A. Samuel, Phys. Rev. D 54 (1996) 6989.

[8] P.A. Rączka, Phys. Rev. D 46 (1992) R3699.

[9] P.A. Rączka, Z. Phys. C 65 (1995) 481.

[10] I.L. Solovtsov, D.V. Shirkov, hep-ph/9711251.

[11] K.A. Milton, I.L. Solovtsov, V.I. Yasnov, preprint OKHEP-98-01, hep-ph/9802282.

[12] J.D. Bjorken, Phys. Rev. 148 (1966) 1467; Phys. Rev. D 1 (1970) 1376.

[13] J. Ellis, M. Karliner, Phys. Lett. B 341 (1995) 397.

[14] Particle Data Group, R. M. Barnett et al., Phys. Rev. D 54 (1996) 1.

[15] S.A. Larin, J.A.M. Vermaseren, Phys. Lett. B 259 (1991) 315.

[16] W. Wetzel, Nucl. Phys. B 139 (1978) 170.

[17] SMC Collaboration, A. Adams et al., Phys. Lett. B 396 (1997) 338; Phys. Rev D 56 (1997) 5330.

[18] SMC Collaboration, A. Adams et al., Phys. Lett. B 412 (1997) 414.

[19] E154 Collaboration, K. Abe et. al., Phys. Lett. B 405 (1997) 180.

[20] P.A. Rączka, A. Szymacha, Phys. Rev. D 54 (1996) 3073.

[21] E143 Collaboration, K. Abe et. al., Phys. Rev. Lett. 78 (1997) 815.

[22] E143 Collaboration, K. Abe et. al., hep-ph/9802357.

[23] G. Grunberg, hep-ph/9705290.

[24] M. Maul, E. Stein, L. Mankiewicz, M. Meyer-Hermann, A. Schäfer, hep-ph/9710392.