Physics 6433, Quantum Field Theory
Assignment #12
Due Friday, December 11, 2009

December 2, 2009

1. Work out the general "phase-space" integral

$$\int d\tilde{p}_a\, d\tilde{p}_b\, \delta^{(4)}(p_a + p_b - P) = \frac{d\Omega}{32\pi^2 M^2}\left[M^2 - (m_a + m_b)^2\right]^{1/2} \times \left[M^2 - (m_a - m_b)^2\right]^{1/2},$$

   where $p_a^2 = -m_a^2$, $p_b^2 = -m_b^2$, $P^2 = -M^2$, and $d\Omega$ is the element of solid angle for the relative momentum.

2. From the Klein-Gordon equation show that the single particle flux vector is

$$s^\mu = 2p^\mu\, d\tilde{p}.$$

   Then consider the invariant flux for two such particles,

$$F = [(s_a s_b)^2 - s_a^2 s_b^2]^{1/2}.$$

   Write this out in terms of particle density s^0 and particle velocity $\mathbf{v} = \mathbf{s}/s^0$, and express F in terms of s_a^0, s_b^0, and $(\mathbf{v}_a - \mathbf{v}_b)^2$ and $(\mathbf{v}_a \times \mathbf{v}_b)^2$. What is F if one particle is at rest (stationary target)? Is this reasonable? Now use the formula for the single particle flux vector to show that

$$F = d\tilde{p}_a\, d\tilde{p}_b 2\left[M^2 - (m_a + m_b)^2\right]^{1/2}\left[M^2 - (m_a - m_b)^2\right]^{1/2},$$

   where $M^2 = -(p_a + p_b)^2$.

3. Use the formula derived in class,

$$\frac{d\sigma}{d\Omega} = \frac{|\Gamma^{(4)}|^2}{64\pi^2 M^2},$$

to obtain explicit formulas for the differential cross section, in terms of the center of mass energy M, and the scattering angle θ,

(a) to order g^2, and

(b) to order g^3.