Physics 6433, Quantum Field Theory
Assignment #1
Due Monday, August 31, 2009

August 24, 2009

1. Consider the description of a single nonrelativistic particle in a central potential, $V(r)$. First introduce spherical polar coordinates r, θ, ϕ:

$$
\begin{aligned}
x &= r\sin\theta\cos\phi, \\
y &= r\sin\theta\sin\phi, \\
z &= r\cos\theta.
\end{aligned}
$$

Write the Lagrangian in polar coordinates. What are Lagrange's equations? Find the canonical momenta p_r, p_θ, p_ϕ. Write the Hamiltonian and the three Hamilton's equations. What is the physical significance of p_θ? Compute $\{p_\theta, H\}$.

2. Prove the following identities concerning Poisson brackets:

$$\{A, BC\} = \{A, B\}C + B\{A, C\}, \tag{1}$$

$$\{A, \{B, C\}\} + \{B, \{C, A\}\} + \{C, \{A, B\}\} = 0. \tag{2}$$

3. A *vector* is an object which transforms in the same way as the position vector $\mathbf{r}$ under a rotation. Therefore, the infinitesimal change in a vector $\mathbf{V}$ under an infinitesimal rotation of the coordinate system is $\delta\mathbf{V} = \delta\boldsymbol{\omega} \times \mathbf{V}$. If $\mathbf{V}$ is constructed from $\mathbf{r}$ and $\mathbf{p}$, show that

$$\delta\mathbf{V} = \{\mathbf{L}\cdot\delta\boldsymbol{\omega}, \mathbf{V}\},$$

where we have used the Poisson bracket notation, and $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. [Hint: It may help to write $\mathbf{V}$ in the general form

$$V = a\mathbf{r} + b\mathbf{p} + c\mathbf{r} \times \mathbf{p},$$

where a, b, and c are functions of the scalars r, p, and $\mathbf{r} \cdot \mathbf{p}$.]

4. The generator of *dilatations* is defined by

$$D = \sum_i q_i p_i.$$

If A is a homogeneous function of the coordinates of degree n, i.e., if

$$A(\lambda q_i) = \lambda^n A(q_i),$$

show that

$$\{D, A\} = nA,$$

where again $\{\,,\}$ represents the Poisson bracket.

5. Verify that when we consider a rigid rotation

$$\delta\mathbf{r} = \delta\boldsymbol{\omega} \times \mathbf{r}$$

in classical particle electrodynamics, the corresponding generator is

$$G = \delta\boldsymbol{\omega} \cdot \mathbf{J},$$

where the angular momentum is

$$\mathbf{J} = \sum_k \mathbf{r}_k \times m_k \mathbf{v}_k + \frac{1}{c} \int (d\mathbf{r})\, \mathbf{r} \times (\mathbf{E} \times \mathbf{B}).$$