Physics 5583. Electrodynamics II.
Midterm Examination I
Spring 2007

March 7, 2007

Instructions: This examination consists of three problems. If you get stuck on one part, assume a result and proceed onward. Do not hesitate to ask questions. GOOD LUCK!

1. This problem has to do with the intrinsic angular momentum associated with a system containing both electric and magnetic charge. Consider a particle of electric charge e fixed at the origin, and a particle carrying magnetic charge g fixed at the point $\mathbf{R}$.
   (a) What are the electric and magnetic fields at all points in space?
   (b) What is the momentum density due to this static configuration of charges?
   (c) What is the angular momentum density due to this static configuration of charges?
   (d) Write down an integral expressing the total angular momentum $\mathbf{J}$ of this system. Does this integral converge? Do not evaluate the integral, but express the dependence of $\mathbf{J}$ on e, g, and $\mathbf{R}$, the vector distance between the electric and magnetic charges. This is the essential basis for the quantization of electric and magnetic charges.

2. One of the continuing conundrums of electrodynamics is the question of whether a uniformly accelerated charge radiates. Consider a particle undergoing constant acceleration **a** for a finite period of time T,

$$\mathbf{v}(t) = \begin{cases} 0, & t \le 0, \\ \mathbf{a}t, & 0 \le t \le T, \\ \mathbf{a}T, & T \le t. \end{cases}$$

   (a) Using the formula for the total radiation due to electric dipole radiation, Larmor's formula, calculate the total power radiated by this particle.

   (b) Instead, we could use the formula derived in class for the power radiated by electric dipole radiation,

$$P = -\frac{1}{6\pi c^3} \dot{\mathbf{d}} \cdot \dddot{\mathbf{d}}. \tag{1}$$

   Compute the derivatives of the electric dipole moment **d** appearing here, to compute the power radiated as a function of time.

   (c) By integrating the power over all time, determine the total energy radiated. Does the result agree with that found in part 2a?

   (d) Comment on the statement: A uniformly accelerated particle radiates only because it is not uniformly accelerated.

3. Consider a particle of charge e moving in a circle of radius r with angular velocity ω. Use either the Larmor formula or Eq. (1) to compute the total power radiated by such a particle. This might be called cyclotron radiation. When relativistic effects are included, the radiation is greatly enhanced, and the result is called synchrotron radiation.