Additional Problems

Due Friday, November 2

1. Starting from the definitions of the Fourier transforms in time:

$$\begin{aligned} \mathbf{E}(\omega) &= \int_{-\infty}^{\infty} dt\, e^{i\omega t}\mathbf{E}(t) \ , \\ \mathbf{H}^*(\omega) &= \int_{-\infty}^{\infty} dt\, e^{-i\omega t}\mathbf{H}(t) \ . \end{aligned}$$

   prove the complex Poynting vector theorem

$$\boldsymbol{\nabla} \cdot (\mathbf{E} \times \mathbf{H}^*) = i\omega \left(\mu|\mathbf{H}|^2 - \varepsilon|\mathbf{E}|^2\right) \ ,$$

   and the energy theorem (ignoring any dependence of ε and μ on the frequency)

$$\boldsymbol{\nabla} \cdot \left(\frac{\partial \mathbf{E}}{\partial \omega} \times \mathbf{H}^* + \mathbf{E}^* \times \frac{\partial \mathbf{H}}{\partial \omega}\right) = i\left(\varepsilon|\mathbf{E}|^2 + \mu|\mathbf{H}|^2\right) \ ,$$

   for waves propagating in a medium without sources.

2. Prove directly from the equations of motion that the total angular momentum is conserved,

$$\frac{d}{dt}\mathbf{L} = \mathbf{0}.$$

Problems in *Classical Electrodynamics*: Chapter 7: 2, 6, 7, 8; Chapter 8: 1, 2.