# Assignment 6
# Due Friday, October 26

October 11, 2007

Problems in *Classical Electrodynamics:* Chapter 6: 1, 2, 3
Additional Problems:

1. Charge $Ze$ is stationed at the origin. A particle of mass $m$ and charge $-e$ is in motion around the stationary charge, as described by the Hamiltonian
$$H = \frac{p^2}{2m} - \frac{Ze^2}{r}.$$
Write the Hamiltonian equations of motion and use them to verify directly the conservation of $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. Now evaluate the time derivative of $\mathbf{r}/r$, and recognize that it can be exhibited in terms of $\mathbf{L}$ and $\mathbf{r}/r^3$. Then conclude that
$$\mathbf{A} = \frac{\mathbf{r}}{r} - \frac{1}{mZe^2}\mathbf{p} \times \mathbf{L},$$
the Runge-Lenz vector, is conserved.

2. In place of the model for ferromagnetism considered in Sec. 6.4, suppose that atomic magnetic moments can have *three* options: parallel, anti-parallel, and perpendicular to the driving field. Find the analogues of Eqs. (6.58), (6.59) for this model by showing that permanent magnetization can exist, with a suitable definition of $T_c$, for $T < T_c$.

3. Show (very simply) that $\mathbf{B}$, the magnetic field of a uniformly moving charge, is related to the electric field $\mathbf{E}$ produced by that charge by $\frac{1}{c}\mathbf{v} \times \mathbf{E}$. Then consider two charges, moving with a common velocity $\mathbf{v}$

along parallel tracks, and show that the magnetic force between them is *opposite* to the electric force, and smaller by the factor $v^2/c^2$. (This is an example of the fact that like charges repel, while like currents attract.) Can you derive the same result by Lorentz transforming the equations of motion in the common rest frame of the two charges? (Hint: coordinates perpendicular to the line of relative motion are unaffected by the transformation.)

4. Charge $e$ is uniformly distributed within a sphere of radius $a$, that is at rest. Evaluate its electrical energy. If the uniform charge distribution now moves at velocity $\mathbf{v}$ ($|\mathbf{v}| \ll c$) determine its magnetic energy and its electromagnetic momentum.