Physics 5573. Electrodynamics I.
First Examination
Fall 2007

October 16, 2007

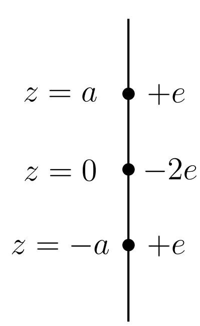
Instructions: This examination consists of three problems. If you get stuck on one part, assume a result and proceed onward. Show all your work. Do not hesitate to ask questions. GOOD LUCK!

1. Consider a configuration of static point charges as shown in the figure.

   (a) Calculate the total charge, the electric dipole moment, and the electric quadrupole moment, defined by the dyadic

   $$\mathbf{q} = \int (d\mathbf{r})\, \rho(\mathbf{r})[3\mathbf{r}\mathbf{r} - \mathbf{1}r^2],$$

   where ρ is the charge density for this configuration of charges. In particular, compute all 9 components of $\mathbf{q}$. Is there a relationship between the components?



(b) Now suppose the system of charges is exposed to a static electric field, which varies only slightly over the system of charges. That is, let a be very small compared to the typical scale of variation of $\mathbf{E}$. Compute the force exerted on the system of charges by this electric field in terms of the total charge, the dipole moment, and the quadrupole moment.

(c) A general formula for the force exerted on an electric quadrupole by a field $\mathbf{E}$ is

$$\mathbf{F} = -\boldsymbol{\nabla} U, \quad U = -\frac{1}{6}\boldsymbol{\nabla}\cdot\mathbf{q}\cdot\mathbf{E}.$$

Show that the result found in part 1b agrees with this formula. [Hint, use the Maxwell's equations appropriate to electrostatics.]

2. The stress dyadic is

$$\mathbf{T} = \mathbf{1}\frac{E^2+B^2}{8\pi} - \frac{\mathbf{EE}+\mathbf{BB}}{4\pi}.$$

(a) By using Maxwell's equations, prove directly that

$$\boldsymbol{\nabla}\cdot\mathbf{T} = -\mathbf{f} - \frac{\partial}{\partial t}\mathbf{G},$$

and determine the form of the force density $\mathbf{f}$ and the momentum density of the electromagnetic field $\mathbf{G}$.

(b) Now consider two point charges of strength e and q respectively, fixed in position, and separated by a distance R. For concreteness, assume e is located at the origin, and q at $z = R$. What is the electric field at any point in space?

(c) For the static situation, show that we can calculate the force on charge q from the surface integral

$$\mathbf{F}_q = -\oint_S d\mathbf{S}\cdot\mathbf{T},$$

where S is an arbitrary volume entirely containing the charge q but not e. By choosing S to be a very small sphere centered on q, use this formula to calculate all components of $\mathbf{F}$. Is the result as expected? [Hint: In integrating over solid angles, show that

$$\oint d\Omega\cos\theta = 0, \quad \oint d\Omega\cos^2\theta = \frac{4\pi}{3}.]$$

3. An idealized plane wave propagating in the z direction can be described by the following electric and magnetic fields,

$$\mathbf{E} = \hat{\mathbf{x}} \text{Re}\, E_0 e^{i(kz-\omega t)},$$
$$\mathbf{B} = \hat{\mathbf{y}} \text{Re}\, B_0 e^{i(kz-\omega t)},$$

in terms of complex amplitudes E_0 and B_0.

(a) From Maxwell's equations, what can you say about the relation between E_0 and B_0, and between k and ω?

(b) Calculate the momentum per unit area carried by this electromagnetic field. [Recall Problem 2a.] Average this over one cycle to get a time-independent result.

(c) Now imagine that this wave is incident upon a perfectly conducting plate perpendicular to the z axis. Describe the reflected wave in a form similar to that given above.

(d) Using the result of part 3b what is the momentum transfered to the plate as a result of the reflection process?