Physics 5573. Electrodynamics I.
First Examination
Fall 2003

October 14, 2003

Instructions: This examination consists of three problems. If you get stuck on one part, assume a result and proceed onward. Show all your work. Do not hesitate to ask questions. GOOD LUCK!

1. Suppose the OU experiment had found a magnetic monopole with magnetic charge g and mass $M = 300$ GeV. Consider the motion of an electron of mass $m \ll M$ and electric charge e in the vicinity of this monopole. Assume the motion of the electron remains nonrelativistic.

   (a) Derive the equation of motion of the electron in the field of the monopole, located at the center of coordinates, in terms of e, g, m, and $\mathbf{r}$, the position vector of the electron relative to the monopole.

   (b) From this equation, derive the equation of energy conservation. What can you say about the speed of the electron?

   (c) From the equation of motion, derive the equation for the time rate of change of the orbital angular momentum of the electron, $L = m\mathbf{r} \times \mathbf{v}$. Deduce the presence of an extra contribution to the angular momentum of the system. How does that contribution depend on the distance between the electron and the monopole?

2. In class we discussed the effect of thermal fluctuations on the orientation of an electric dipole in an external electric field. We used a classical Boltzmann distribution for this purpose. However, it is more accurate

to recognize that angular momentum is quantized, and consequently, that the possible values of the electric dipole moment along the direction of the electric field take on only a discrete set of values. Consider the simplest case, that of a spin-1/2 molecule, where the possible values of $\mathbf{d}$ along the z direction, the direction of the electric field, are $\pm d$.

(a) Calculate $\langle d_z \rangle$ as a function of d, E, and T.

(b) What does $\langle d_z \rangle$ become when dE/kT is small?

(c) Calculate the electric susceptibility corresponding to a gas of such molecules, with n molecules per unit volume.

3. (a) Using Maxwell's equations, evaluate the rate of change of field energy,

$$\frac{\partial}{\partial t} U, \quad U = \frac{E^2 + B^2}{8\pi}.$$

Identify the resulting expression in terms of the rate at which work is done on the charges, and the flux of energy leaving a given volume.

(b) Prove the identity

$$\int_V (d\mathbf{r}) |\mathbf{E} \times \mathbf{B}| \leq \int_V (d\mathbf{r}) \frac{E^2 + B^2}{2},$$

true for any volume V.

(c) Use this result, and the relation between the energy flux vector $\mathbf{S}$ derived in part 3a and the field momentum density $\mathbf{G}$,

$$\mathbf{S} = c^2 \mathbf{G},$$

to show that the velocity $\mathbf{v}$ of a field configuration completely enclosed in a fixed volume V, which contains no charges or currents, defined by

$$\mathbf{v} = \frac{\frac{d}{dt} \int_V (d\mathbf{r})\, \mathbf{r}\, U}{\int_V (d\mathbf{r}) U},$$

is always bounded by the velocity of light,

$$|\mathbf{v}| \leq c.$$