Electrodynamics I
First Assignment
Due Friday, February 5

January 16, 2016

1. Use the three-dimensional Levi-Civita symbol, ϵ_{ijk}, which has the properties of being totally antisymmetric in all three indices,

$$\epsilon_{ijk} = \epsilon_{jki} = \epsilon_{kij} = -\epsilon_{jik} = -\epsilon_{kji} = -\epsilon_{ikj},$$

and is normalized to unity,

$$\epsilon_{123} = +1$$

to define the cross product in terms of the Cartesian components of vectors:

$$\mathbf{A} = \mathbf{B} \times \mathbf{C} \Leftrightarrow A_i = \epsilon_{ijk} B_j C_k,$$

which uses the summation convention for repeated indices, for example,

$$A_i B_i = \sum_{i=1}^{3} A_i B_i = \mathbf{A} \cdot \mathbf{B}.$$

Prove the identity

$$\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl},$$

where δ_{ij} is the Kronecker delta symbol,

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

Show this is equivalent to the familiar identity

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}).$$

provided **A**, **B**, and **C** are commuting vectors.

2. Show that the following functions represent the delta function in one dimension by showing that

$$\delta_\epsilon(x \neq 0) \to 0 \quad \text{as} \quad \epsilon \to 0+,$$

and

$$\int_{-\infty}^{\infty} dx\, \delta_\epsilon(x) = 1.$$

(a)

$$\delta_\epsilon(x) = \frac{1}{\sqrt{\pi\epsilon}} e^{-x^2/\epsilon},$$

(b)

$$\delta_\epsilon(x) = \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2},$$

(c)

$$\delta_\epsilon(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikx - |k|\epsilon}.$$

Problems in *Classical Electrodynamics*, Chapter 1: 2, 4, 5; Chapter 2: 1, 2; Chapter 3: 1, 2.