1.a $\vec{F} = e\left[\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right] = m\frac{d^2\vec{r}}{dt^2} = m\frac{d\vec{v}}{dt}$ 3 3 2

$\vec{E} = 0$, $\vec{B} = g\frac{\vec{r}}{r^3}$ 4

so $\boxed{m\frac{d\vec{v}}{dt} = \frac{eg}{c}\frac{\vec{v}\times\vec{r}}{r^3}}$

b) $\vec{v}\cdot m\frac{d\vec{v}}{dt} = 0 = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$

8 $\boxed{E = \frac{1}{2}mv^2 = \text{const}}$ v = speed = const 2

c) $\vec{r}\times m\frac{d\vec{v}}{dt} = m\frac{d}{dt}(\vec{r}\times\vec{v}) - m\overbrace{\vec{v}\times\vec{v}}^{=0}$ 3

$= \frac{eg}{c}\frac{\vec{r}\times(\vec{v}\times\vec{r})}{r^3} = \frac{eg}{c}\left(\frac{\vec{v}}{r} - \frac{\vec{r}\cdot\vec{v}\,\vec{r}}{r^3}\right)$

$= \frac{eg}{c}\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ 6

because $\frac{d}{dt}\frac{1}{r} = -\frac{1}{r^2}\frac{dr}{dt} = -\frac{1}{r^3}r\frac{dr}{dt}$

$[dr^2 = 2r\,dr = 2\vec{r}\cdot d\vec{r}]$ $= -\frac{1}{r^3}\vec{r}\cdot\frac{d\vec{r}}{dt} = \frac{-\vec{r}\cdot\vec{v}}{r^3}$

$$\therefore \frac{d}{dt}\left(\vec{L} - \frac{eg}{c}\frac{\vec{r}}{r}\right) = 0$$

$$\vec{J} = \vec{L} - \frac{eg}{c}\frac{\vec{r}}{r} = \text{const}$$

$\vec{S} = -\frac{eg}{c}\frac{\vec{r}}{r}$ is additional angular momentum due to e-g pair

$|\vec{S}| = \frac{eg}{c}$ is a const, indep. of the dist. between the monopole & the electron.

2.

a) $$\langle d_z \rangle = \frac{d e^{dE/kT} - d e^{-dE/kT}}{e^{dE/kT} + e^{-dE/kT}} = d \tanh \frac{dE}{kT}$$

b) If $dE/kT \ll 1$, $\tanh\frac{dE}{kT} \simeq \frac{dE}{kT}$, and

$$\langle d_z \rangle = \frac{d^2 E}{kT}$$

c) $\vec{P} = n\langle\vec{d}\rangle = \frac{nd^2}{kT}\vec{E}$, $\chi = \frac{nd^2}{kT}$ //

3.

a) $$\frac{\partial}{\partial t}\frac{E^2+B^2}{8\pi} = \frac{1}{4\pi}\left(\vec{E}\cdot\frac{\partial}{\partial t}\vec{E} + \vec{B}\cdot\frac{\partial}{\partial t}\vec{B}\right)$$

$$= \frac{c}{4\pi}\vec{E}\cdot\left(\nabla\times\vec{B} - \frac{4\pi}{c}\vec{j}\right) + \frac{c}{4\pi}\vec{B}\cdot\left(-\vec{\nabla}\times\vec{E}\right)$$

$$= -\vec{E}\cdot\vec{j} + \frac{c}{4\pi}E\cdot\nabla\times\vec{B} - \frac{c}{4\pi}\vec{B}\cdot\vec{\nabla}\times\vec{E}$$

$$= -\vec{E}\cdot\vec{j} + \frac{c}{4\pi}\left[\underbrace{(\vec{\nabla}\times\vec{B})\cdot\vec{E}}_{\nabla_B\cdot\vec{B}\times\vec{E}} - \underbrace{(\vec{\nabla}\times\vec{E})\cdot\vec{B}}_{\nabla_E\cdot\vec{E}\times\vec{B}}\right]$$

$$= -\vec{E}\cdot\vec{j} - \frac{c}{4\pi}\vec{\nabla}\cdot(\vec{E}\times\vec{B})$$

so $$\frac{\partial}{\partial t}\left(\frac{E^2+B^2}{8\pi}\right) + \vec{\nabla}\cdot\frac{c}{4\pi}(\vec{E}\times\vec{B}) + \vec{E}\cdot\vec{j} = 0$$

$U = \frac{E^2+B^2}{8\pi}$ = energy density of field

$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B}$ = energy flux vector

$\vec{E}\cdot\vec{j}$ = rate per unit volume at which work is done on charges

b) $|\vec{E}\times\vec{B}|^2 = E^2B^2\sin^2\theta = E^2B^2(1-\cos^2\theta)$ 4

$$\leq E^2B^2 = \left(\frac{E^2+B^2}{2}\right)^2 - \left(\frac{E^2-B^2}{2}\right)^2$$

$$\leq \left(\frac{E^2+B^2}{2}\right)^2$$

$$\therefore \int_V (d\vec{r})\,|\vec{E}\times\vec{B}| \leq \int_V (d\vec{r})\,\frac{E^2+B^2}{2}$$

c) $\dfrac{\partial U}{\partial t} + \vec{\nabla}\cdot\vec{S} = 0$ in charge-free region

or $\dfrac{\partial \vec{r}U}{\partial t} + \vec{r}\,\vec{\nabla}\cdot\vec{S} = 0$

or $\dfrac{\partial \vec{r}U}{\partial t} + \vec{\nabla}\cdot(\vec{S}\vec{r}) - \underbrace{\vec{S}}_{c^2\vec{G}} = 0$

$$\int_V (d\vec{r})\,\frac{\partial}{\partial t}\vec{r}U = \frac{d}{dt}\int_V (d\vec{r})\,\vec{r}U = c^2\int (d\vec{r})\,\vec{G}$$

$$= c^2\vec{P} \qquad c^2\vec{G} = \frac{c}{4\pi}\vec{E}\times\vec{B}$$

$$\vec{v} = \frac{c^2\vec{P}}{E},\ |\vec{v}| = \frac{\frac{c}{4\pi}\left|\int (d\vec{r})\,\vec{E}\times\vec{B}\right|}{\frac{1}{8\pi}\int (d\vec{r})(E^2+B^2)} \leq \frac{c\int (d\vec{r})\,|\vec{E}\times\vec{B}|}{\frac{1}{2}\int (d\vec{r})(E^2+B^2)} \leq c$$