Physics 5163. Homework 7
Due Monday, April 20, 2009

April 13, 2009

Problems in Pathria: 6.1, 6.22, 6.23, 6.26.

1. Consider a one-dimensional, imperfect gas, consisting of hard, impenetrable "spheres" of length a and mass m. If there are N such particles in a "box" of length L, find the partition function and the equation of state. (For an ideal gas, consisting of interpenetrating points, the latter is $pL = NkT$.)

2. Consider a system of N classical linear molecules with electric dipole moments d. Assume there is an electric field $\mathcal{E}$ pointing in the z direction. The Hamiltonian for this system is

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2m} + \frac{1}{2I} \left(p_{\theta_j}^2 + \frac{p_{\phi_j}^2}{\sin^2 \theta_j} \right) - \mathcal{E} d \cos \theta_j \right].$$

   The instantaneous dipole moment in the z direction is

$$\mathcal{F}_{\mathcal{E}} = -\frac{\partial H}{\partial \mathcal{E}} = \sum_j d \cos \theta_j.$$

   Find the average energy U, the specific heat $c_{\mathcal{E}}$, the average "force" $\langle \mathcal{F}_{\mathcal{E}} \rangle$, and its variance.

3. Consider *fixed*, spin-1/2 particles, with dipole moment d, in an electric field $\mathcal{E}$, where the Hamiltonian is

$$H = -\sum_{j=1}^{N} \mathcal{E} d \sigma_{zj}.$$

   Here σ_{zj} can take on the values ± 1. Find U, $c_{\mathcal{E}}$, $\langle \mathcal{F}_{\mathcal{E}} \rangle$, and its variance.