

# Physics 5163. Homework 7

## Due Wednesday, April 7, 2010

March 30, 2010

**Problems in Pathria:** 6.1, 6.22, 6.23, 6.26.

1. Consider a one-dimensional, imperfect gas, consisting of hard, impenetrable “spheres” of length  $a$  and mass  $m$ . If there are  $N$  such particles in a “box” of length  $L$ , find the partition function and the equation of state. (For an ideal gas, consisting of interpenetrating points, the latter is  $pL = NkT$ .)
2. Consider a system of  $N$  classical linear molecules with electric dipole moments  $d$ . Assume there is an electric field  $\mathcal{E}$  pointing in the  $z$  direction. The Hamiltonian for this system is

$$H = \sum_{j=1}^N \left[ \frac{p_j^2}{2m} + \frac{1}{2I} \left( p_{\theta_j}^2 + \frac{p_{\phi_j}^2}{\sin^2 \theta_j} \right) - \mathcal{E} d \cos \theta_j \right].$$

The instantaneous dipole moment in the  $z$  direction is

$$\mathcal{F}_{\mathcal{E}} = -\frac{\partial H}{\partial \mathcal{E}} = \sum_j d \cos \theta_j.$$

Find the average energy  $U$ , the specific heat  $c_{\mathcal{E}}$ , the average “force”  $\langle \mathcal{F}_{\mathcal{E}} \rangle$ , and its variance.

3. Consider *fixed*, spin-1/2 particles, with dipole moment  $d$ , in an electric field  $\mathcal{E}$ , where the Hamiltonian is

$$H = -\sum_{j=1}^N \mathcal{E} d \sigma_{zj}.$$

Here  $\sigma_{zj}$  can take on the values  $\pm 1$ . Find  $U$ ,  $c_{\mathcal{E}}$ ,  $\langle \mathcal{F}_{\mathcal{E}} \rangle$ , and its variance.