Physics 5163. Homework 6
Due Monday, April 13, 2009

April 6, 2009

1. Consider the structure function $\Omega(E)$ for a general system. Recall the expression for $\Omega(E)$ given in class in the saddle-point approximation, where the contour C is chosen to pass through the stationary point β along the path of steepest descents. Recall further the expression for $\Omega(E)$ when the path crosses the real axis at a different point $\bar{\beta}$. Call the first expression for the structure function Ω_1, the second Ω_2. Show that for an ideal gas of $N \gg 1$ molecules, $\Omega_1/\Omega_2 \approx 1$ provided $|\delta\beta/\beta| \ll \mathcal{O}(N^{-1/3})$, where $\delta\beta = \bar{\beta} - \beta$.

2. Show that, in general, for a system of many components, so that $\ln \chi = \mathcal{O}(N)$, the same result holds.

3. Supply the details of the saddle-point analysis leading the the derivation for the formula of the *quantum* structure function,

$$\Omega_\epsilon(E) = \frac{\epsilon e^{\beta E}\chi(\beta)}{\sqrt{2\pi(\ln(\chi(\beta))''}}.$$

   In particular, show that for the validity of this formula, we must have the following restriction on ϵ:

$$\epsilon \ll \pi\sqrt{N}kT.$$