1. Consider the structure function $\Omega(E)$ for a general system. Recall the expression for $\Omega(E)$ given in class in the saddle-point approximation, where the contour $C$ is chosen to pass through the stationary point $\beta$ along the path of steepest descents. Recall further the expression for $\Omega(E)$ when the path crosses the real axis at a different point $\bar{\beta}$. Call the first expression for the structure function $\Omega_1$, the second $\Omega_2$. Show that for an ideal gas of $N \gg 1$ molecules, $\Omega_1/\Omega_2 \approx 1$ provided $|\delta \beta/\beta| \ll O(N^{-1/3})$, where $\delta \beta = \bar{\beta} - \beta$.

2. Show that, in general, for a system of many components, so that $\ln \chi = O(N)$, the same result holds.

3. Supply the details of the saddle-point analysis leading the the derivation for the formula of the quantum structure function,

$$\Omega_{\epsilon}(E) = \frac{\epsilon e^{\beta E} \chi(\beta)}{\sqrt{2\pi \ln(\chi(\beta))}}.$$  

In particular, show that for the validity of this formula, we must have the following restriction on $\epsilon$:

$$\epsilon \ll \pi \sqrt{N} kT.$$