Physics 5163
Homework 4
Due Wednesday, March 4

February 23, 2009

1. Compute the partition function \( Z(\beta) \) for the classical one-dimensional harmonic oscillator defined by the Hamiltonian

\[
H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2.
\]

Compare the result with that for the quantum harmonic oscillator discussed in class, in the high-temperature limit, \( \beta \to 0 \). Do they agree? What about Planck’s constant \( h \)?

2. Consider a relativistic particle for which the Hamiltonian is

\[
H = c \sqrt{p^2 + m^2 c^2} - m c^2.
\]

(a) Consider one-dimensional motion in an interval of length \( L \). Find the partition function in the extreme relativistic limit, \( p \gg mc \).

(b) Consider the same situation in general. Express your result in terms of the modified Bessel function,

\[
K_\nu(x) = \int_0^\infty e^{-z \cosh \theta} \cosh \nu \theta \ d\theta.
\]

Look up the behavior of \( K_\nu(z) \) for small \( z \) to reproduce the result of part b.

(c) Consider a relativistic particle in three dimensions, where it is confined to a box of volume \( V \). Find the partition function in the extreme relativistic limit.
(d) Repeat part c in general.

Problems in Pathria: 3.17, 3.18, 3.24, and 3.30.