Physics 5163
Homework 4
Due Wednesday, March 4

February 23, 2009

1. Compute the partition function $Z(\beta)$ for the *classical* one-dimensional harmonic oscillator defined by the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2.$$

Compare the result with that for the quantum harmonic oscillator discussed in class, in the high-temperature limit, $\beta \to 0$. Do they agree? What about Planck's constant h?

2. Consider a relativistic particle for which the Hamiltonian is

$$H = c\sqrt{p^2 + m^2c^2} - mc^2.$$

   (a) Consider one-dimensional motion in an interval of length L. Find the partition function in the extreme relativistic limit, $p \gg mc$.

   (b) Consider the same situation in general. Express your result in terms of the modified Bessel function,

$$K_\nu(x) = \int_0^\infty e^{-z\cosh\theta}\cosh\nu\theta\, d\theta.$$

   Look up the behavior of $K_\nu(z)$ for small z to reproduce the result of part b.

   (c) Consider a relativistic particle in three dimensions, where it is confined to a box of volume V. Find the partition function in the extreme relativistic limit.

(d) Repeat part c in general.

Problems in Pathria: 3.17, 3.18, 3.24, and 3.30.