1. Derive the result
\[ \frac{\Gamma\left(\frac{3N}{2}\right)}{\Gamma\left(\frac{3N-1}{2}\right)} \approx \left(\frac{3N}{2}\right)^{1/2}, \quad N \to \infty \]
using the Stirling approximation for the Gamma function. What order in \( N \) is the correction to this leading asymptotic approximation?

2. Using the microcanonical ensemble, derive, for an ideal gas, the distribution function
\[ \mathcal{P}(p_1, p_2, p_3) \]
for the three Cartesian momentum components of a single molecule.

3. Starting from the expression for the volume of an \( n \)-dimensional sphere of radius \( r \),
\[ V_n(r) = \frac{\pi^{n/2}}{\Gamma(n/2 + 1)} r^n, \]
calculate the difference between the volumes of two spheres of slightly different radii,
\[ V_n(r) - V_n(r - t), \]
in the limit
\[ n \gg 1, \quad \frac{t}{r} \ll \left(\frac{2}{n}\right)^{1/2} \ll 1. \]
Show that if \( n \sim 10^{22}, \frac{t}{r} \sim 10^{-22} \), the shell so defined contains all but about 1/3 of the volume, and just a bit more would include it all!
4. Assume
\[ \rho(q,p) = \text{const.} \times \psi_E(H), \]
where \( \psi_E(H) \) is the characteristic function,
\[ \psi_E(H) = \begin{cases} 
1, & \text{if } E > H, \\
0, & \text{if } E < H, 
\end{cases} \]
for the \( N \)-particle gas. According to Problem 3, nearly all the volume of the energy sphere lies very, very close to the surface. Show that the single momentum distribution \( \mathcal{P}(p_1) \) is \textit{still} the Maxwell distribution.

\textbf{Problems from Pathria:} 5.2, 5.3.