Physics 5163
Homework 3
Due Tuesday, February 17

February 9, 2009

1. Derive the result
$$\frac{\Gamma\left(\frac{3N}{2}\right)}{\Gamma\left(\frac{3N-1}{2}\right)} \approx \left(\frac{3N}{2}\right)^{1/2}, \quad N \to \infty$$
using the Stirling approximation for the Gamma function. What order in N is the correction to this leading asymptotic approximation?

2. Using the microcanonical ensemble, derive, for an ideal gas, the distribution function
$$\mathcal{P}(p_1, p_2, p_3)$$
for the three Cartesian momentum components of a single molecule.

3. Starting from the expression for the volume of an n-dimensional sphere of radius r,
$$V_n(r) = \frac{\pi^{n/2}}{\Gamma(n/2+1)} r^n,$$
calculate the difference between the volumes of two spheres of slightly different radii,
$$V_n(r) - V_n(r-t),$$
in the limit
$$n \gg 1, \quad \frac{t}{r} \ll \left(\frac{2}{n}\right)^{1/2} \ll 1.$$
Show that if $n \sim 10^{22}$, $t/r \sim 10^{-22}$, the shell so defined contains all but about 1/3 of the volume, and just a bit more would include it all!

4. Assume

$$\rho(q,p) = \text{const.} \times \psi_E(H),$$

where $\psi_E(H)$ is the characteristic function,

$$\psi_E(H) = \begin{cases} 1, & \text{if } E > H, \\ 0, & \text{if } E < H, \end{cases}$$

for the N-particle gas. According to Problem 3, nearly all the volume of the energy sphere lies very, very close to the surface. Show that the single momentum distribution $\mathcal{P}(p_1)$ is *still* the Maxwell distribution.

Problems from Pathria: 5.2, 5.3.