Statistical Mechanics
Physics 5163
Homework Assignment #2
Due Friday, February 6, 2009

January 27, 2009

These problems all refer to the classical microcanonical distribution, where

$$\Omega(E) = \int \delta(E - H)\, dq\, dp.$$

1. Evaluate $\Omega(E)$ for a free particle, $H = p^2/2m$, in one dimension, confined to a box of length L. Show that

   $$\Omega(E) = L\left(\frac{2m}{E}\right)^{1/2}.$$

   What is $\Omega(E)$ for a free particle confined to a three-dimensional box of volume V?

2. A one-dimensional harmonic oscillator is described by the Hamiltonian

   $$H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2).$$

   Show that

   $$\Omega(E) = \frac{2\pi}{\omega}.$$

3. Generalize this result by showing that for any confined one-dimensional system we always have

   $$\Omega(E) = \text{period of motion}.$$

4. A harmonic oscillator in three dimensions is described by the Hamiltonian

$$H = \frac{1}{2m}(\mathbf{p}^2 + m^2\omega^2\mathbf{r}^2).$$

Find $\Omega(E)$.

5. For a linear molecule, with moment of inertia I, for which the Hamiltonian is

$$H = \frac{p^2}{2m} + \frac{1}{2I}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right),$$

confined in a box of volume V, find $\Omega(E)$.

6. If the molecule in the previous problem is put in an electric field $\mathcal{E}$, the Hamiltonian becomes

$$H = \frac{1}{2I}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + \mathcal{E}d(1 - \cos\theta),$$

where d is the electric dipole moment of the molecule, the z axis is the direction of the electric field, and we have ignored the translational degree of freedom. Find $\Omega(E)$ for *all* energies.

Problems from Pathria: 5.2, 5.3.