Physics 5163
Statistical Mechanics
First Examination

March 6, 2009

Instructions: Work both problems and show all your steps. If you have any questions, ask the instructor. If you get stuck on one part, assume an answer to continue on. Remember, this is a closed-book examination, so you may not consult books or notes. **Good Luck!**

1. Consider a one-dimensional harmonic oscillator, with Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2.$$

This problem uses the canonical distribution.

(a) Compute the partition function, $Z(\beta)$, classically.

(b) Compute the partition function, $Z(\beta)$, quantum mechanically.

(c) Compute the mean energy, $U = \langle H \rangle$, classically.

(d) Compute the mean energy, $U = \langle H \rangle$, quantum mechanically. When is the classical value recovered? What about the equipartition theorem?

(e) Compute the fluctuation in the energy, $\langle (H-U)^2 \rangle$, classically, and

(f) quantum mechanically.

(g) What is the specific heat at constant volume, c_v, in the classical and quantum mechanical cases?

2. Consider a classical nonrelativistic particle moving in a linear potential in three dimensions, described by the Hamiltonian ($\lambda > 0$)

$$H = \frac{p^2}{2m} + \lambda r,$$

where $p^2 = p_x^2 + p_y^2 + p_z^2$ and $r = \sqrt{x^2 + y^2 + z^2}$.

   (a) Compute the structure function $\Omega(E)$.

   (b) Compute the partition function $Z(\beta)$.

   (c) Verify by explicit integration that these quantities are related by a Laplace transformation:

$$Z(\beta) = \int_0^\infty dE\, e^{-\beta E} \Omega(E).$$

   (d) From the partition function, compute the average energy U.

   (e) Is the above result consistent with the generalized equipartition theorem,

$$\langle p_i \frac{\partial H}{\partial p_i} \rangle = \langle q_i \frac{\partial H}{\partial q_i} \rangle = kT \quad \text{(no sum on } i\text{)?}$$