

Second Examination
Physics 5013, Mathematical Methods of
Physics

November 18, 2011

Instructions: Attempt all parts of this exam. If you get stuck on one part, assume an answer and proceed on. Do not hesitate to ask questions. Remember this is a closed book, closed notes, exam. *Good luck!*

These problems involve evaluating integrals using the residue theorem. In each case state carefully relation of the integral in question to a closed contour integral, state the contour of integration carefully, state where the poles are, and evaluate the residue of the poles.

1. Evaluate the integral

$$\int_0^{\infty} dx \frac{\sin x}{x} \frac{1}{(x^2 + a^2)^2}, \quad a > 0,$$

using Jordan's lemma. Be careful about $x = 0$.

2. Evaluate the integral

$$\int_0^{\infty} dx \frac{\sqrt{x}}{x^2 + 1},$$

using the residue theorem. Be careful about the phase associated with the square root.

3. Convert

$$\int_0^{2\pi} d\theta \frac{1}{(\cos \theta + a)}, \quad a > 1,$$

to a contour integral, and evaluate the latter by the residue theorem.