Physics 5013. Homework 6
Due Friday, November 3, 2006

October 24, 2006

Problems in Bender and Orzag:

Chapter 8, pp. 412–3: **18, 19, 20, 31**

Additional problems:

1. Consider the function
$$\frac{1}{z}\log(1+z).$$
Derive the $[3,3]$ Padé approximant stated in class
$$P_3^3(z) = \frac{1 + \frac{17}{14}z + \frac{1}{3}z^2 + \frac{1}{140}z^3}{1 + \frac{12}{7}z + \frac{6}{7}z^2 + \frac{4}{35}z^3}.$$
Similarly, work out $P_4^3(z)$, and verify the values given in class for $z = 0.5$, 1, and 2.

2. The Stirling series for the Gamma function is for $n \to \infty$,
$$\Gamma(n) = (n-1)! \sim \left(\frac{2\pi}{n}\right)^{1/2}\left(\frac{n}{e}\right)^n\left(1 + \frac{A_1}{n} + \frac{A_2}{n^2} + \frac{A_3}{n^3} + \frac{A_4}{n^4} + \dots\right),$$
where
$$\begin{aligned} A_1 &= \frac{1}{12}, \\ A_2 &= \frac{1}{288}, \\ A_3 &= -\frac{139}{51840}, \\ A_4 &= -\frac{571}{2488320}. \end{aligned}$$

Compute the $[1,1]$ and $[2,2]$ Padé approximants for $\Gamma(x)(e/x)^x\sqrt{x/2\pi}$. Compare numerically the values so obtained with the exact function for $x = 0.2$, 0.5, and 1.0, which are *small* values of x. Can a more accurate approximation be obtained by averaging P_1^1 and P_2^2?

3. Consider a continued-fraction representation of the exponential function in the form

$$e^x = \frac{c_0}{1 + \frac{c_1 z}{1+\frac{c_2 z}{1+\frac{c_3 z}{1+\ldots}}}}.$$

Show that

$$c_0 = -c_1 = 1, \quad c_{2n} = \frac{1}{4n-2}, \quad c_{2n+1} = -\frac{1}{4n+2}, \quad n \geq 1.$$

How many terms must be included to compute e to 8 significant figures?