Physics 5013. Homework 2
Due Friday, September 9, 2011

September 2, 2011

Problems in Whittaker and Watson: Problems 1, 2, and 3, Chapter III, page 59.

1. For what range of positive values of x is

$$\sum_{n=0}^{\infty} \frac{1}{1+x^n}$$

   (a) Convergent?

   (b) Uniformly convergent?

2. In numerical analysis it is often convenient to replace derivatives by finite differences. For example, we might represent the second derivative of a function as follows:

$$\frac{d^2}{dx^2}\psi(x) \approx \frac{1}{h^2}[\psi(x+h) - 2\psi(x) + \psi(x-h)].$$

   Regarding h as a small parameter, find the error in this approximation.

3. Compute e from the Taylor series of e^x about 0 to 16 significant figures.

4. Given that

$$\int_0^1 \frac{dx}{1+x^2} = \tan^{-1} x\Big|_0^1 = \frac{\pi}{4},$$

   expand the integrand into a series and integrate term by term, obtaining

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \cdots + (-1)^n \frac{1}{2n+1} + \cdots,$$

which is Leibnitz's formula for π (actually discovered by Gregory in 1671). This formula converges so slowly that it is quite useless for numerical work: compute the first 100 terms and see for yourself!