Physics 5013. Homework 1
Due Friday, September 2, 2011

August 22, 2011

Problems in Whittaker and Watson: Chapter II, Problems 2, 6, and 10, pp. 38–39.

1. Prove that
$$\lim_{n\to\infty}(\sqrt{n+1}-\sqrt{n})(\sqrt{n+\frac{1}{2}}) = \frac{1}{2}.$$

2. Let $a_n = \frac{10^n}{n!}$. (a) To what limit does a_n converge as $n \to \infty$? (b) Is the sequence monotonic? (c) Is it monotonic from a certain n onwards? (d) Give an estimate of the difference between a_n and the limit. (e) From what value of n onwards is this difference less than $\frac{1}{100}$?

3. Prove that the sequence $\sqrt{2}$, $\sqrt{2\sqrt{2}}$, $\sqrt{2\sqrt{2\sqrt{2}}}$, $\cdots$, converges. Find its limit.

4. For what real values of α does
$$\sum_{n=1}^{\infty} \frac{n!}{(1+\alpha)(2+\alpha)\cdots(n+\alpha)}$$
converge?

5. Test for convergence:

   (a)
$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

(b)

$$\sum_{n=1}^{\infty} \log(1 + \frac{1}{n})$$

(c)

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

(d)

$$\sum_{n=1}^{\infty} \frac{1}{n\, n^{1/n}}$$

(e)

$$\sum_{n=1}^{\infty} \frac{1}{n\, 2^n}$$

6. For what real values of p and q will the following series converge?

$$\sum_{n=2}^{\infty} \frac{1}{n^p (\ln n)^q}$$