Introduction to Quantum Mechanics II
Quiz 9

Name:

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We can construct the spin-1 states from combining two spin-1/2 systems from the bottom up. Assume that

$$|j=1, m=-1\rangle = |m_1 = -1/2, m_2 = -1/2\rangle.$$

Then by applying the *raising* operator,

$$\frac{1}{\hbar} J_+ |jm\rangle = \sqrt{(j-m)(j+m+1)}|jm+1\rangle$$

compute

$$|j=1, m=0\rangle \quad \text{and} \quad |j=1, m=1\rangle$$

in terms of the $|m_1, m_2\rangle$ states for the individual spin 1/2's, by successively applying J_+ to $|j=1, m=-1\rangle$ and to $|j=1, m=0\rangle$.

Solution:

$$\frac{1}{\hbar} J_+ |1,-1\rangle = \sqrt{2}|1,0\rangle = \frac{1}{2}(\sigma_{1+} + \sigma_{2+})|--\rangle = |-+\rangle + |+-\rangle,$$

so

$$|1,0\rangle = \frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle).$$

Then,

$$\begin{aligned}\frac{1}{\hbar} J_+ |1,0\rangle = \sqrt{2}|1,1\rangle &= \frac{1}{2}(\sigma_{1+} + \sigma_{2+})\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle) \\ &= \frac{1}{\sqrt{2}}(|++\rangle + |++\rangle) = \sqrt{2}|++\rangle,\end{aligned}$$

or

$$|1,1\rangle = |++\rangle.$$