Introduction to Quantum Mechanics II
Quiz 7

Name:

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Consider $\mathbf{r}, \mathbf{p}$ variables satisfying

$$[r_k, p_l] = i\hbar\delta_{kl}.$$

The orbital angular momentum is defined by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}.$$

Show that

$$\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p}$$

is Hermitian, but

$$\mathbf{p} \times \mathbf{L} + \mathbf{L} \times \mathbf{p}$$

is not. Evaluate the latter in terms of $\mathbf{p}$. [Hint: it will suffice to consider a single component of these equations, such as the z component.

Solution:

$$(\mathbf{p} \times \mathbf{L})_z = p_x L_y - p_y L_x, \quad (\mathbf{L} \times \mathbf{p})_z = L_x p_y - L_y p_x,$$

so since $\mathbf{p}$ and $\mathbf{L}$ are both Hermitian,

$$(\mathbf{p} \times \mathbf{L})^\dagger = -\mathbf{L} \times \mathbf{p}.$$

Therefore $\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p}$ is Hermitian, while

$$(\mathbf{p} \times \mathbf{L} + \mathbf{L} \times \mathbf{p})^\dagger = -\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p}.$$

In fact,

$$(\mathbf{p} \times \mathbf{L} + \mathbf{L} \times \mathbf{p})_z = [p_x, L_y] - [p_y, L_x] = i\hbar p_z - (-i\hbar p_z) = 2i\hbar p_z,$$

or in general, since which direction we call the z axis is arbitrary,

$$\mathbf{p} \times \mathbf{L} + \mathbf{L} \times \mathbf{p} = 2i\hbar \mathbf{p}.$$