Introduction to Quantum Mechanics II
Quiz 6

Name:

September 28, 2012

Given the commutators,

$$[R_k, R_l] = 0, \quad [P_k, P_l] = 0, \quad [R_k, P_l] = i\hbar\delta_{kl},$$

and the constructions for the angular momentum and the boost generator,

$$\mathbf{J} = \mathbf{R} \times \mathbf{P} + \mathbf{S}, \quad \mathbf{N} = \mathbf{P}t - M\mathbf{R},$$

and the fact that the spin $\mathbf{S}$ is independent of the coordinates,

$$[S_k, R_l] = [S_k, P_l] = 0,$$

evaluate

$$[\mathbf{R}, \delta\boldsymbol{\omega} \cdot \mathbf{J}], \quad [\mathbf{P}, \delta\boldsymbol{\omega} \cdot \mathbf{J}],$$

and

$$[\mathbf{R}, \delta\mathbf{v} \cdot \mathbf{N}], \quad [\mathbf{P}, \delta\mathbf{v} \cdot \mathbf{N}].$$

Solution:

$$\frac{1}{i\hbar}[\mathbf{R}, \delta\boldsymbol{\omega} \times \mathbf{R} \cdot \mathbf{P}] = \delta\boldsymbol{\omega} \times \mathbf{R} \cdot \frac{1}{i\hbar}[\mathbf{R}, \mathbf{P}] = \delta\boldsymbol{\omega} \times \mathbf{R},$$

$$\frac{1}{i\hbar}[\mathbf{P}, -\delta\boldsymbol{\omega} \times \mathbf{P} \cdot \mathbf{R}] = -\delta\boldsymbol{\omega} \times \mathbf{P} \cdot \frac{1}{i\hbar}[\mathbf{P}, \mathbf{R}] = \delta\boldsymbol{\omega} \times \mathbf{P},$$

$$\frac{1}{i\hbar}[\mathbf{R}, \delta\mathbf{v} \cdot \mathbf{P}t] = \delta\mathbf{v}t, \quad \frac{1}{i\hbar}[\mathbf{P}, -M\mathbf{R} \cdot \delta\mathbf{v}] = M\delta\mathbf{v}.$$