Introduction to Quantum Mechanics II
Quiz 5

Name:

September 21, 2012

In terms of harmonic oscillator variables satisfying

$$[q, p] = i,$$

compute

$$f(q') = e^{iq'p} q e^{-iq'p},$$

where q' is a number, by differentiating $f(q')$ with respect to q'. Then, evaluate

$$e^{iq'p} q^n e^{-iq'p},$$

where n is an integer. Then, by expanding the exponential in a power series, show that

$$e^{iq'p} e^{ip'q} e^{-iq'p} = e^{ip'(q+q')}.$$

Solution:

$$\frac{\partial}{\partial q'} f(q') = e^{iq'p} i(pq - qp) e^{-iq'p} = 1,$$

so $f(q') = q + q'$. Then

$$e^{iq'p} q^n e^{-iq'p} = e^{iq'p} q e^{-iq'p} e^{iq'p} q e^{-iq'p} \cdots e^{iq'p} q e^{-iq'p} = (q + q')^n.$$

Now for any function which can be expanded in a power series,

$$g(q) = \sum_{n=0}^{\infty} a_n q^n,$$

we have

$$e^{iq'p} g(q) e^{-iq'p} = \sum_{n=0}^{\infty} a_n e^{iq'p} q^n e^{-iq'p} = \sum_{n=0}^{\infty} a_n (q + q')^n = g(q + q').$$