Introduction to Quantum Mechanics II
Quiz 4

Name:

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The states $|n\rangle$ of the harmonic oscillator are eigenstates of the energy operator or Hamiltonian,

$$H = \frac{p^2 + q^2}{2}$$

What are the eigenvalues of H in these states (don't derive this, just give the answer):

$$H|n\rangle = E_n|n\rangle, \quad E_n = ?$$

From

$$\langle q'|H|n\rangle = E_n\langle q'|n\rangle,$$

obtain the "time-independent Schrödinger equation" or energy eigenvalue equation satisfied by $\psi_n(q') = \langle q'|n\rangle$. Show for the ground state, $n = 0$, this equation is satisfied if

$$\psi_0(q') = e^{-\frac{1}{2}q'^2}.$$

Solution: $E_n = n + \frac{1}{2}$, so

$$\langle q'|\frac{p^2 + q^2}{2}|n\rangle = \frac{1}{2}\left(-\frac{d^2}{dq'^2} + q'^2\right)\psi_n(q') = \left(n + \frac{1}{2}\right)\psi_n(q').$$

For $n = 0$,

$$\frac{d}{dq'}e^{-\frac{1}{2}q'^2} = -q'e^{-\frac{1}{2}q'^2},$$

so

$$\frac{d^2}{dq'^2}e^{-\frac{1}{2}q'^2} = (-1 + q'^2)e^{-\frac{1}{2}q'^2},$$

so

$$\frac{1}{2}\left(-\frac{d^2}{dq'^2}+q'^2\right)\psi_0(q')=\frac{1}{2}\psi_0(q').$$