Introduction to Quantum Mechanics II
Quiz 3

Name:

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Using the realization of the position and momentum operators on eigenstates of position,

$$\langle q'|q = q'\langle q'|, \quad \langle q'|p = \frac{1}{i}\frac{\partial}{\partial q'}\langle q'|,$$

compute

$$\langle q'|[q,p].$$

Is this consistent with the canonical commutator

$$[q,p] = i?$$

If we assume that on an eigenstate of momentum, similarly,

$$\langle p'|p = p'\langle p'|, \quad \langle p'|q = \alpha\frac{\partial}{\partial p'}\langle p'|,$$

compute

$$\langle p'|[q,p].$$

What must α be for this to be consistent with the same canonical commutation relation?

Solution:

$$\begin{aligned}
\langle q'|[q,p] &= q'\langle q'|p - \frac{1}{i}\frac{\partial}{\partial q'}\langle q'|q = q'\frac{1}{i}\frac{\partial}{\partial q'}\langle q'| - \frac{1}{i}\frac{\partial}{\partial q'}q'\langle q'| \\
&= \left(q'\frac{1}{i}\frac{\partial}{\partial q'} - \frac{1}{i}\frac{\partial}{\partial q'}q'\right)\langle q'| = i\langle q'|.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\langle p'|[q,p] &= \alpha\frac{\partial}{\partial p'}\langle p'|p - p'\langle p'|q = \alpha\frac{\partial}{\partial p'}p'\langle p'| - p'\alpha\frac{\partial}{\partial p'}\langle p'| \\
&= \left(\alpha\frac{\partial}{\partial p'}p' - p'\alpha\frac{\partial}{\partial p'}\right)\langle p'| = \alpha\langle p'| = i\langle p'|,
\end{aligned}$$

so $\alpha = i$.