Introduction to Quantum Mechanics II
Quiz 11

Name:

November 9, 2012

The general rotation operator is given in terms of Eulerian angles ϕ, θ, ψ by

$$U(\phi,\theta,\psi) = e^{i\psi J_z/\hbar} e^{i\theta J_y/\hbar} e^{i\phi J_z/\hbar}.$$

Compute for $j = 1$ the 3×3 rotation matrix

$$\langle jm|U(\phi,\theta,\psi)|jm'\rangle$$

for the special case $\theta = 0$. This Euler rotation carries the z axis to a new $\bar{z}$ axis. What is the direction of $\bar{z}$? For this special case of the rotation matrix, compute the probabilities $p(m,\bar{z};m',z)$, that is, the probability of finding the value of $J_{\bar{z}}$ to be $m\hbar$ given that the state was prepared in the state where J_z had the value $m'\hbar$. Why is this expected?

Solution: Quite generally,

$$\langle jm|e^{i\psi J_z/\hbar} e^{i\theta J_y/\hbar} e^{i\phi J_z/\hbar}|jm'\rangle = e^{im\psi}\langle jm|e^{i\theta J_y/\hbar}|jm'\rangle e^{im'\phi}.$$

If $\theta = 0$, the two successive rotations are about the z axis, $\bar{z} = z$. The rotation matrix is

$$U(\phi,0,\psi) = \begin{pmatrix} e^{i(\psi+\phi)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i(\psi+\phi)} \end{pmatrix}.$$

The probabilities are $p(m,m') = \delta_{mm'}$, which is obvious since the second measurement simply confirms the first.