Introduction to Quantum Mechanics II
Quiz 10

Name:

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In this problem, consider the hydrogen atom, but include the spin of the electron, a spin-1/2 particle, but disregard the spin of the nucleus. Also, ignore the effect of the electron spin on the energies of the states of the atom. Consider the principal quantum number $n = 2$ set of states. How many such states are there? What are the possible values of the angular momentum quantum number l? By considering the possible values of l, count the number of states with $n = 2$. Does it agree with the count of states you found before? Now add the angular momentum $\mathbf{L}$ and the electron spin $\mathbf{S}$, $\mathbf{J} = \mathbf{L} + \mathbf{S}$. What are the possible values of $\mathbf{J}^2/\hbar^2 = j(j+1)$? Count the number of states with $n = 2$ in terms of j. Does that agree with the counts above?

Solution: Not counting spin, there are n^2 states. There are 2 possible spin states for each n, l, m value, so that means for $n = 2$ there are $2 \times 4 = 8$ states. In terms of angular momentum, the possible values of l are 0 and 1, and there is one orbital angular momentum state with $l = 0$ and three with $l = 1$; multiplying by the number of spin states give

$$2 \times (1 + 3) = 8 \,\text{states}.$$

Or, in terms of the total angular momentum quantum number j, which takes on the values $1 + \frac{1}{2} = \frac{3}{2}$ and $1 - \frac{1}{2} = 0 + \frac{1}{2} = \frac{1}{2}$, there are

$$\left(2 \times \frac{3}{2} + 1\right) + \left(2 \times \frac{1}{2} + 1\right) \times 2 = 4 + 4 = 8$$

states, where the last factor of 2 comes from the fact that $j = 1/2$ can arise either from $l = 0$ (which gives only $j = 1/2$) or $l = 1$.