Introduction to Quantum Mechanics II
Quiz 14

Name:

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The Zeeman effect is due to the interaction of the magnetic dipole moment of the atom with an external magnetic field,

$$\Delta E = -\boldsymbol{\mu} \cdot \mathbf{B}.$$

For the orbital motion of the electron,

$$\boldsymbol{\mu} = \frac{e}{2mc}\mathbf{L},$$

where m is the mass of the electron, and $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ is the orbital angular momentum. Consider the $n = 3$ states of the hydrogen atom. How many such states are there? Label the states by the angular momentum and magnetic quantum numbers, l and m, respectively. Let the direction of the external magnetic field define the z axis. What is the shift in energy of these states due to the magnetic field? Express your answer in terms of the Bohr magneton, $\mu_B = \frac{e\hbar}{2mc}$. How many distinct energy levels are there belonging to principal quantum number $n = 3$? Which state has the lowest energy?

Solution: There are $n^2 = 9$ $n = 3$ states, labeled by

$$\{|nlm\rangle\} = \{|300\rangle, |311\rangle, |310\rangle, |31,-1\rangle, |322\rangle, |321\rangle, |320\rangle, |32,-1\rangle, |32,-2\rangle\}.$$

The energy shift is

$$\Delta E = -\frac{eB}{2m_e c}L_z = -\mu_B B m,$$

in terms of the magnetic quantum number m. Since m takes on the values 2, 1, 0, -1, -2, there are 5 energy levels. The lowest energy level is that for $m = -2$ taking into account that the charge on the electron is negative.